OFDM

Exercise 1: Are the magnitude and phase of this signal even or odd?

Let $S(f) = \mathcal{F}\{\sin(2\pi f_c t)\} = \frac{1}{2j}\left(\delta(f - f_c) - \delta(f + f_c)\right)$

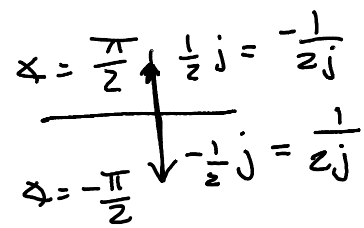
$$|S(f)| = \tfrac{1}{2}\delta(f - f_c) + \tfrac{1}{2}\delta(f + f_c)$$

$$|S(-f)| = \tfrac{1}{2}\delta(-f - f_c) + \tfrac{1}{2}\delta(-f + f_c)$$

but $\delta(-f - f_c) = \delta(f_c + f)$ (only non-zero at $f = -f_c$)

$\delta(-f + f_c) = \delta(f_c - f)$ (" $f = f_c$)

$\therefore |S(-f)| = |S(f)| \rightarrow$ the magnitude is even



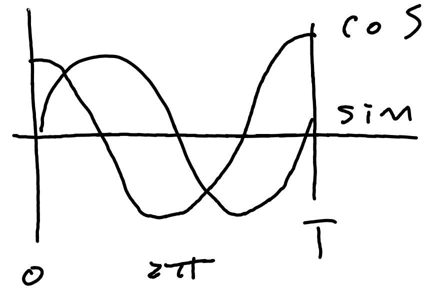
$$\measuredangle S(f) = \begin{cases} -\frac{\pi}{2} & \text{at } f = f_c \\ +\frac{\pi}{2} & \text{at } f = -f_c \end{cases}$$

$$\measuredangle S(-f) = \measuredangle\left(\frac{1}{2j}\delta(f_c + f) - \frac{1}{2j}\delta(f_c - f)\right)$$

$$= \begin{cases} \frac{\pi}{2} & \text{at } f = f_c \\ -\frac{\pi}{2} & \text{at } f = -f_c \end{cases}$$

$= -\measuredangle S(f) \rightarrow$ the phase is odd.

Exercise 2: Show this.

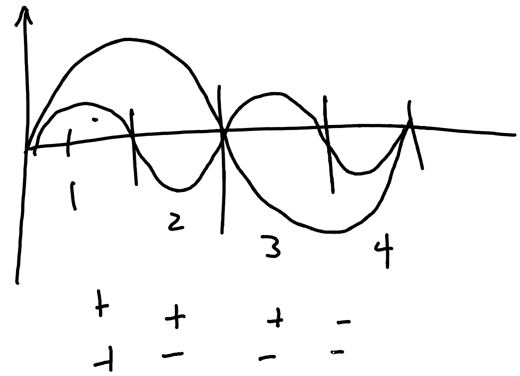


$f_2 = n f_1 \qquad f_1 = \frac{1}{T}$

$$\int_0^{\frac{1}{f_1} = T} \cos 2\pi f_1 t \; \cos 2\pi n f_1 t \; dt$$

$= 0$ for n an integer.

$$\int_0 \cos\theta \; \sin\theta \; d\theta = 0$$

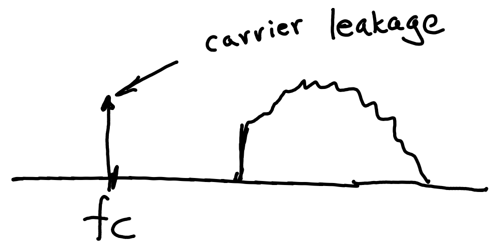


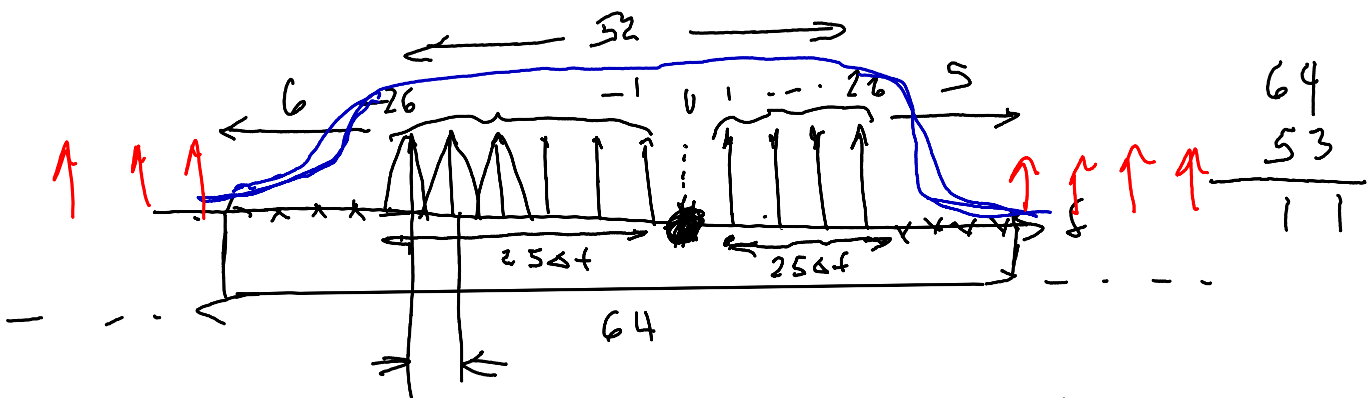
Exercise 3: The 802.11g WLAN standard uses OFDM with a sampling rate of 20 MHz, with $N = 64$. What is the subcarrier frequency spacing?

symbol duration is: $T = \underbrace{64}_{\text{number of samples}} \cdot \underbrace{\frac{1}{20\,\text{MHz}}}_{\text{sample period}}$

frequency spacing: $\frac{1}{T} = \frac{20}{64}$ MHz $=$ **312.5 kHz**

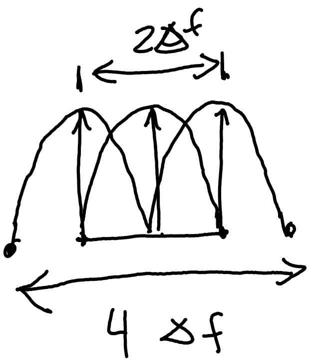
Exercise 4: The 802.11g specification uses 52 of the $N = 64$ possible subcarriers and omits both the DC (zero frequency) and the highest-frequency subcarriers. What is the bandwidth of the signal?

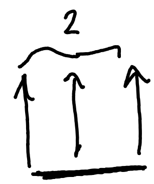




64
53
11

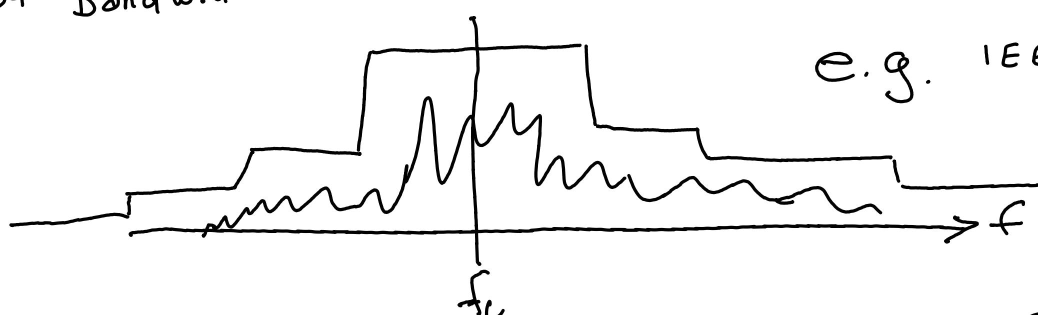
$$\frac{20\text{ MHz}}{64} \cdot 52 = 16.25\text{ MHz}$$





- 3dB ←
- 99% power ←
- - X dB down.

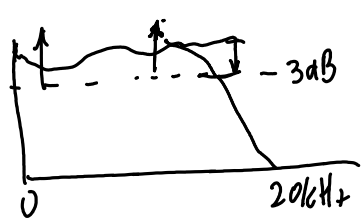
Different definitions of Bandwidth



e.g. IEEE specification for spectrum mask

99.1% of power (e.g. FCC)

< 0.1%



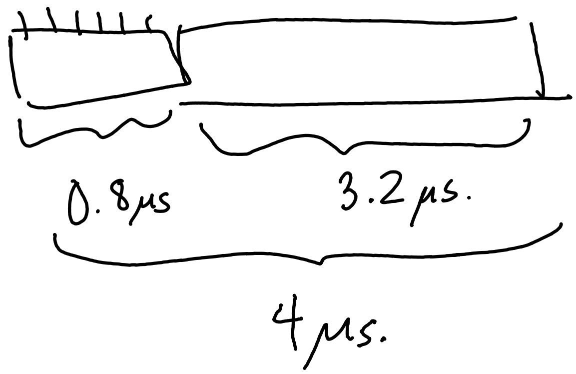
(e.g. audio quality)

Exercise 5: The 802.11g WLAN standard uses OFDM with a sampling rate of 20 MHz, with $N = 64$ and guard interval of $0.8\mu s$. What is the total duration of each OFDM block, including the guard interval? How long is the guard time?

$$T_S = \frac{1}{20\text{MHz}} = 50\text{ns.}$$

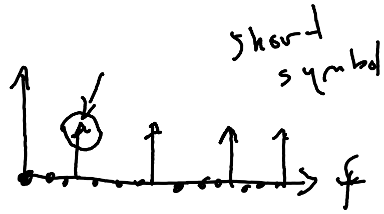
many samples

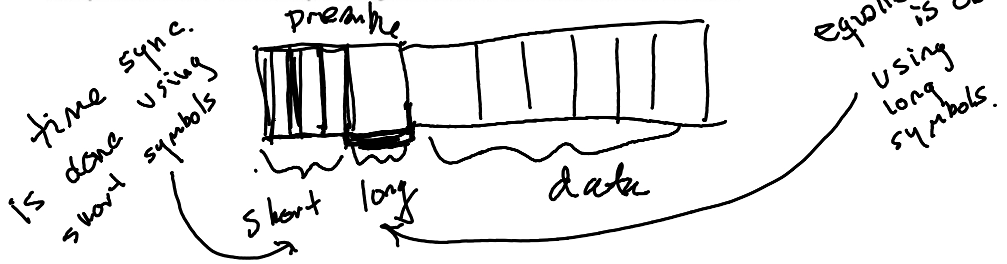
N=64 @ 20MHz

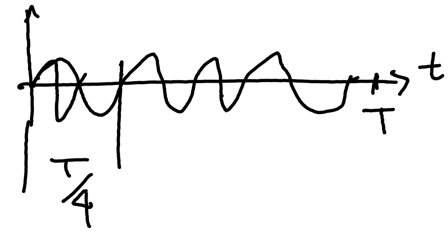


$$\frac{0.8\,\mu s}{50\text{ ns/sample}} = \frac{80}{50}\text{ samples} = \underline{\underline{16}}\text{ samples.}$$

Exercise 6: The 802.11g preamble contains a "short" followed by a "long" training symbol. The short symbol contains only every fourth subcarrier. What is the period of this symbol? The long training symbol contains fixed data on each of the data subcarriers. How would you use the long training symbol to correct the phase and amplitude of subsequently-received data subcarriers?

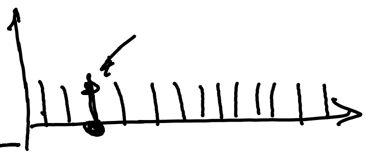




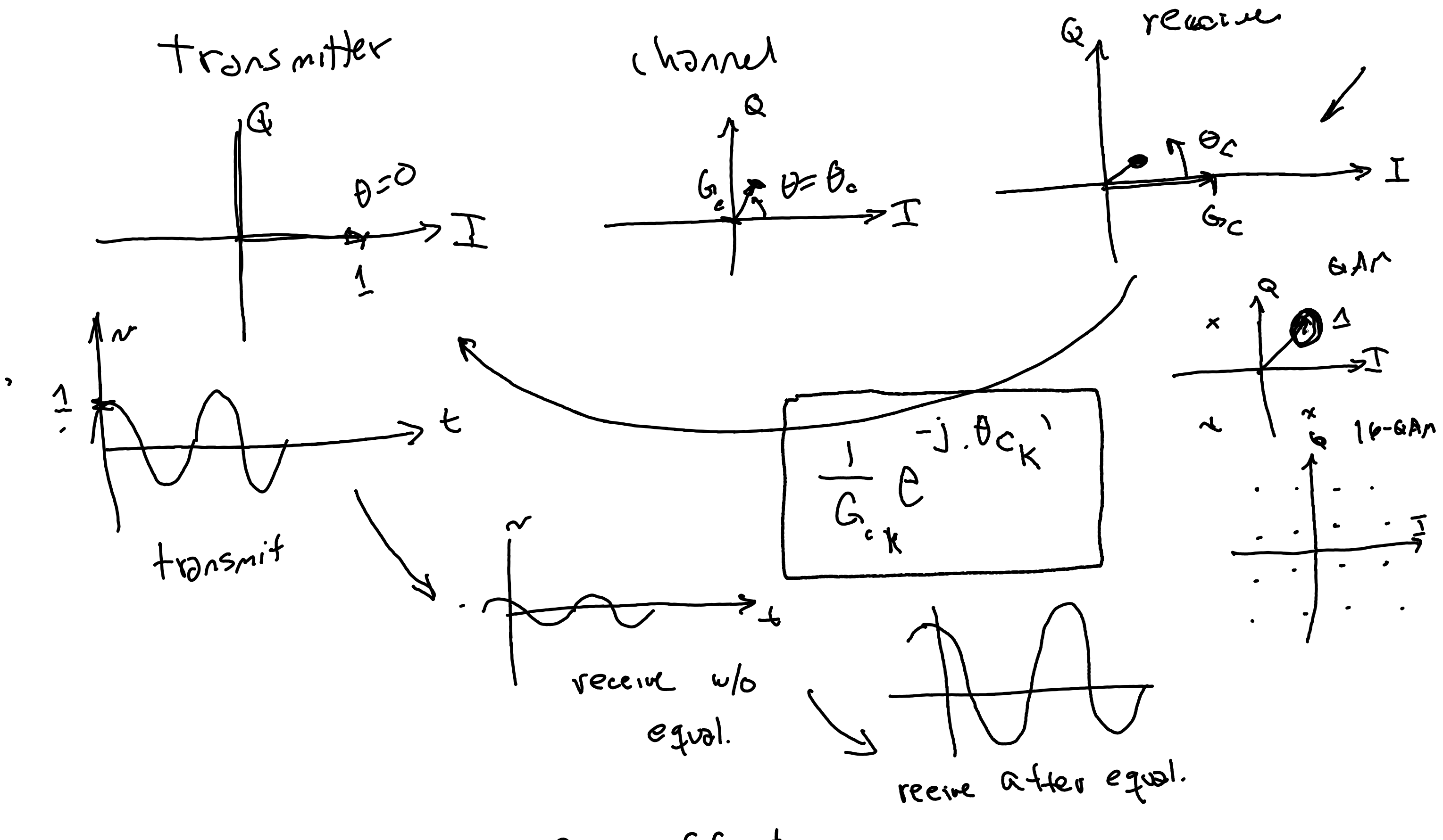


if use only every 4th subcarrier

period is 1/4 of symbol duration

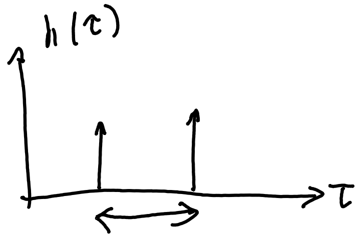


(e.g. if T = 3.2μs period of short symbol is 0.8μs).



to correct for effect of channel: divide by channel gain & subtract channel phase shift.

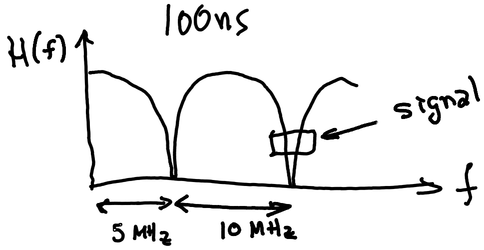
Exercise 7: A channel's impulse response is two equal-level impulses separated by 100 ns. What difference in propagation path lengths would result in such an impulse response? How far apart are the nulls of this channel? What OFDM signal bandwidth(s) would be required to provide frequency diversity?



$c = 300\ m/\mu s$

$d = c\,t = 300 \cdot 0.1 = 30\ m$

need a delay equal to one period or a phase shift of 2π between nulls.



$100ns = \frac{1}{\Delta f}$

$\Delta f = \frac{1}{100ns} = 10 MHz$

$\tau\ \frac{1}{2\Delta f} = \frac{1}{200ns}$ (first null at 5 MHz = 180° phase shift).

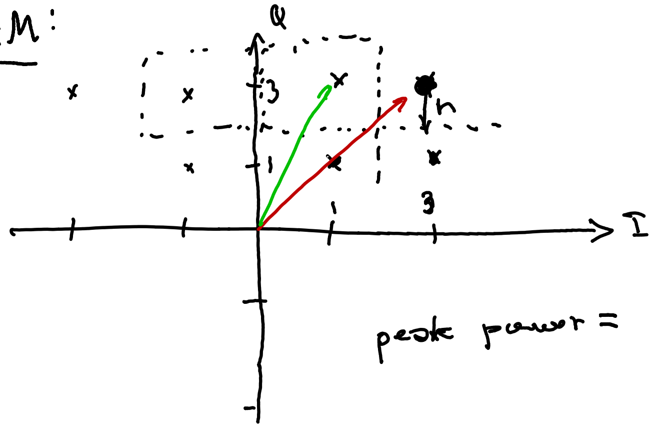
rule of thumb: bandwidth $> \frac{1}{\text{delay spread}}$

(real channel impulse response is more complex)

Exercise 8: How many bits per subcarrier are transmitted by an OFDM system using 16-QAM? Assuming equal noise powers, how much more power does this system need to achieve the same BER than a system using 4-QAM? Than a system using BPSK?

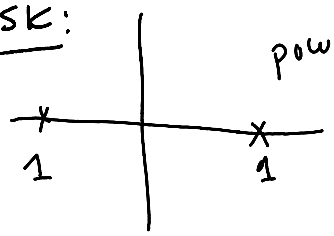
16-QAM:

$\log_2(16) = 4$ bits per subcarrier



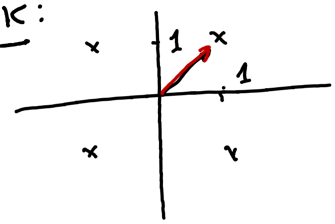
peak power $= 3^2 + 3^2 = 18$

BPSK:



power = 1

QPSK:



power = 2

BUT: these are the powers of individual subcarriers.

OFDM signal amplitude has Rayleigh p.d.f.

so subcarrier powers don't directly determine the RF power.