Diversity

Exercise 1: Which of these might lead to a reduction in system efficiency by requiring more time or bandwidth? Which of these would require additional or more complex antennas?

	need more time or freq	higher complexity
Space	NO	YES
Freq	YES (FREQ)	YES
Time	YES (TIME)	SOMEWHAT
Polarization	NO	YES

Exercise 2: What spacing is required for 10λ separation at 900 MHz?

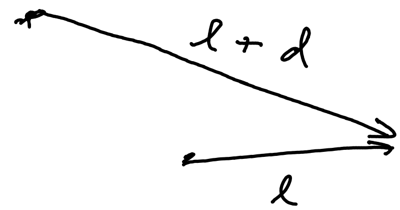
900 MHz

$10\lambda =$

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{9 \times 10^8} = 33\,\text{cm.}$$

$$10\lambda \approx 3.3\,m$$

Exercise 3: What time difference results from a path length difference of d m? If the frequency is f, What is the resulting phase difference? What difference in f is required for two equal-gain paths to cancel each other? How far apart would the frequency nulls be for a channel with two equal-gain paths with path lengths that differ by 300m? By 30m? What time delay differences does this correspond to?



time difference: $t = \frac{d}{c}$

$\cos(\underbrace{2\pi f t}_{\theta})$ ← $2\pi f t$ is the phase

phase difference: $\underline{\Delta\theta} = t \cdot 2\pi f = \underbrace{\frac{2\pi f d}{c}}_{\text{phase difference.}}$

delay

$$\boxed{\begin{array}{l} \text{e.g. } \underline{d = 150\,m} \quad c = \underline{300\,m/\mu s} \\ \Delta\theta = 2\pi f \frac{d}{c} = 2\pi f \frac{150}{300} = \pi f \end{array}}$$

for complete cancellation the frequencies are:

$$\overbrace{\frac{2\pi f_n d}{c}} = (2n+1)\pi$$

$$f_n = \frac{(2n+1)\cancel{\pi} c}{2\cancel{\pi} d}$$

$\Delta\theta = \pi$

$f_n = 0.5(2n+1) = n + \frac{1}{2}$

$f_{n+1} = 0.5(2(n+1)+1) = n + 1 + \frac{1}{2}$

$f_{n+1} - f_n = 1$

$f_n = \frac{3 \times 10^8}{2 \cdot 300} = 0.5(2n+1)\,MHz$

for $d = 300\,m$ spacing is $f_{n+1} - f_n = 1\,MHz$

for $d = 30\,m$ $f_n = 5(2n+1)\ MHz$

spacing is 10 MHz

Exercise 4: Would time diversity be more or less effective as the receiver's speed increased? What would happen if the receiver was stopped (such as a traffic light)?

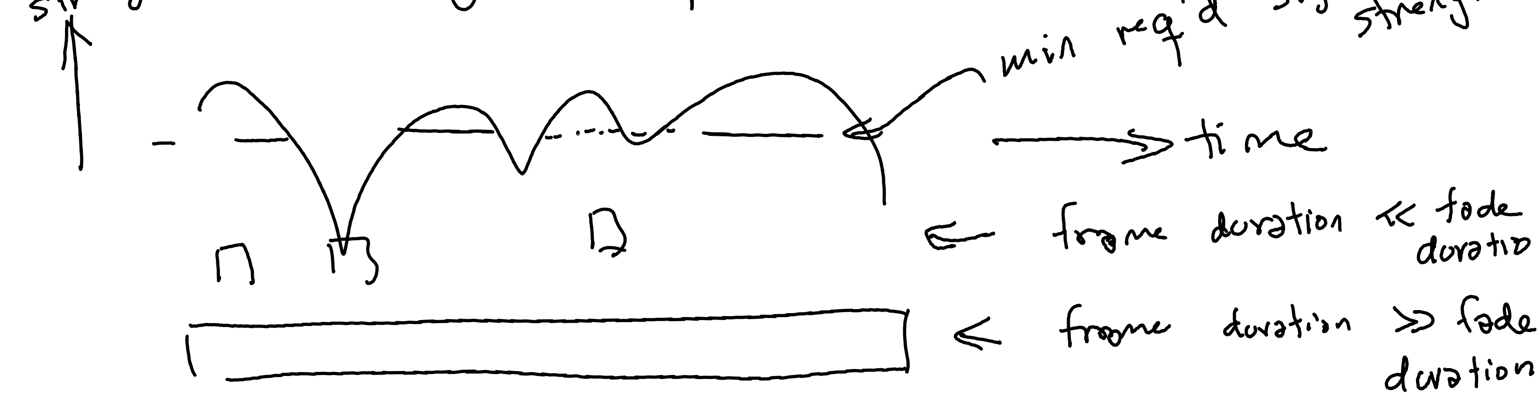
faster → less time between fades →

less delay

until fades are ≈ duration of frames

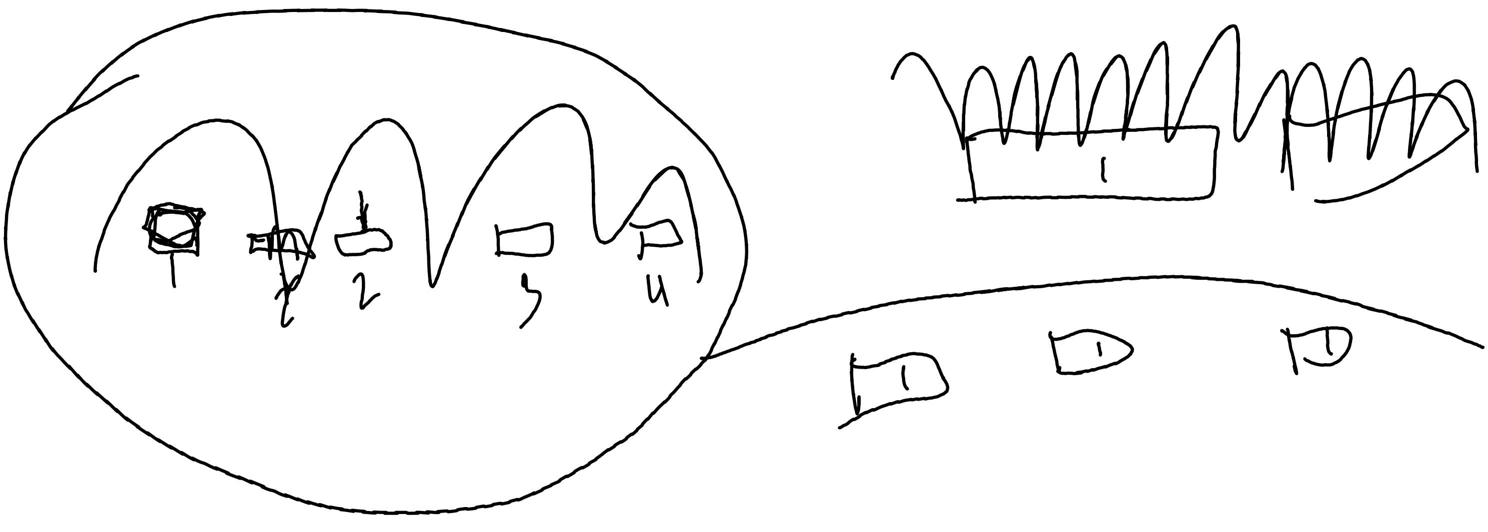
then less likely to receive

a complete frame



if stopped → no change in fading

→ time diversity not effective.



x_1 & x_2 are noise (independent r.v.)

$$E[(x_1+x_2)^2] = \underline{E[x_1^2]} + \cancel{E[2x_1x_2]}^{\,0} + \underline{E[x_2^2]}$$

the power of sum of two independent r.v. is sum of their powers.

$$z = x_1 x_2 \qquad E\{z\} = \sum p(z)\, z$$

		x_2: -1	$+1$
x_1	-1	$\frac{1}{4}$	$\frac{1}{4}$
	$+1$	$\frac{1}{4}$	$\frac{1}{4}$

$$E[x_1 x_2] = \sum P(x_1, x_2)\,(x_1 x_2)$$

$$= \frac{1}{4}(-1\cdot -1) + \frac{1}{4}(-1\cdot +1) + \frac{1}{4}(+1\cdot -1) + \frac{1}{4}(+1\cdot +1)$$

$$= \frac{1}{4}(1-1-1+1) = \frac{1}{4}(0) = 0.$$

indepence:

$$P(x,y) = P(x)\,P(y)$$

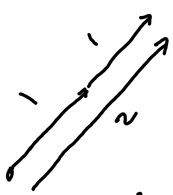
Exercise 5: Assuming maximal-ratio combining, what would be the resulting SNR if the branch SNRs were +10 dB and +20 dB? If they were both +10 dB?

10 dB + 20 dB = ~~30 dB~~. wrong

$$10^{\left(\frac{10}{10}\right)} + 10^{\left(\frac{20}{10}\right)} = 10 + 100 = 110 \approx 20\text{dB}.$$

10 dB & 10 dB

$$10^{\frac{10}{10}} + 10^{\frac{10}{10}} = 10 + 10 = 20 = 13\text{ dB}.$$



$(1+1)^2 = 2^2 = 4$

signal voltage doubled: increased power by 6 dB

noise power doubled. increase by 3dB

net increase in SNR = 6 − 3 = 3dB.

Exercise 6: Assuming independent Rayleigh fading, the same SNRs as in the previous exercise and that the signal is considered "faded" if the SNR is below 0 dB, what fraction of time would be signal be faded with and without two-branch selection diversity?

10 dB   20 dB

— 0 dB

p (fade) = 10%   p = 1%

$p(\text{both faded}) = 0.1 \times 0.01 = 10^{-3} = 0.1\%$

If 10 dB & 10 dB : $p\left(\begin{smallmatrix}\text{both}\\\text{faded}\end{smallmatrix}\right) = \left(10^{-1}\right)^2 = 1\%$

Exercise 7: What type of diversity would you expect to be implemented in an (inexpensive) WLAN card? In a cellular base station?

- cheap: switching
- base station: maximal ratio.