Statistical Models of Average Path Loss

Statistical Path Loss Models

To compute the loss between a transmitter and receiver using ray tracing would require measuring the positions and properties of all objects that could interact with the radio waves as they propagated between the two. This would have to be done to an accuracy much less than a wavelength. This is clearly impractical in real-world situations.

Instead, we treat the path loss as a random (unpredictable) value and try to predict its average value as a function of distance for various other parameters such as frequency, antenna heights, type of environment (urban vs. rural vs. suburban), building heights, street width, and many others.

It's important to understand that the actual path loss measured in any location can differ significantly, by perhaps ±10dB, from the predicted average value. This is true for even the best statistical path loss models.

These path loss models were derived by measuring the path loss as a function of distance in specific locations. Initially the measured path loss values were recorded in tables or plotted. Curves were then fit to these measurements to derive the models.

Much work has been put into refining these propagation models because they are useful for predicting the coverage of wireless systems for both design and deployment.

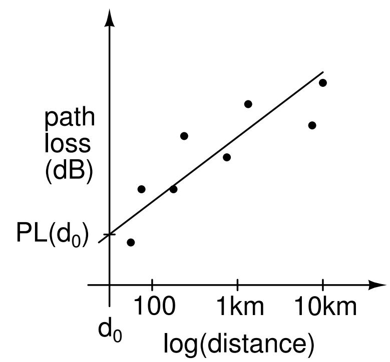
Power Law Model

The simplest model assumes the path loss increases as a power, n, of the distance:

$$PL(d) = \left(\frac{d}{d_0}\right)^n$$

For example, $n = 2$ for free-space propagation. This equation may only be accurate for a certain range of distances.

We can obtain the exponent for NLOS conditions by measuring the path loss in dB versus distance, plotting the path loss versus the log of distance and fitting a straight line:



The equation of this line is:

$$PL(d)_{\text{dB}} = PL(d_0) + 10n \log\left(\frac{d}{d_0}\right)$$

where d_0 is a reference distance (e.g. 1 m) and $10n$ is the slope of the line. For free-space propagation n is 2. But in NLOS environments n is larger, typically about 3 for outdoor cellular systems and higher for indoor propagation.

Exercise 1: What is the free-space path loss, in dB, at 10 m for $f = 1500$ MHz? What is the value of $PL(1\text{ km})$?

Exercise 2: If the path loss is 90 dB at 100 m and 120 dB at $d = 1$ km, what are n and $PL(d_0 = 1\text{ m})$?

Exercise 3: What path would you have to travel if you wanted to measure the average path loss at a given distance from a particular transmitter?

It's also possible to fit two or more straight lines to the path loss measurements. Typically, this is because the propagation mechanism changes at a certain distance (e.g. from LOS to NLOS).

Okumura-Hata Model

We can improve the predictive property of the simple power-law path loss model by including additional parameters such as antenna heights, frequency and type of terrain.

Models of this type were developed by Okumura who made extensive measurements in Japan in the

1960's. He plotted best-fit loss versus distance curves for various combinations of parameters. Hata later derived equations that provided the same numerical results without having to look up values in graphs. A European committee, COST-231, extended the Hata model to frequencies above 1500 MHz. These models have been further refined over time.

As an example, the Okumura-Hata model predicts the path loss in urban areas as[1]:

$$L_b = 69.55 + 26.16 \cdot \log \frac{f}{\text{MHz}} - 13.82 \cdot \log \frac{h_{\text{Base}}}{\text{m}} - a(h_{\text{Mobile}})$$
$$+ (44.9 - 6.55 \cdot \log \frac{h_{\text{Base}}}{\text{m}}) \cdot \log \frac{d}{\text{km}}$$

where:

$$a(h_{\text{Mobile}}) = (1.1 \cdot \log \frac{f}{\text{MHz}} - 0.7) \frac{h_{\text{Mobile}}}{\text{m}} - (1.56 \cdot \log \frac{f}{\text{MHz}} - 0.8)$$

The model is restricted to:

f:	150 ... 1000 MHz
h_{Base}:	30 ...200 m
h_{Mobile}:	1 ...10 m
d:	1 ...20 km

1) "log" means "$\log_{10}$"

Exercise 4: Compute the median path loss predicted by the Okumura-Hata model at $f = 900$MHz, base station and mobile antenna heights of 30m and 1m respectively, and a distance of 2km.

There are various other models that attempt to improve prediction accuracy by using additional parameters. For example, the Walfish-Bertoni model is used for small cells with low base station heights and uses street widths and building heights as additional parameters.

These models are only valid over the parameter values represented in the original measurements. Extrapolating beyond these values may not produce accurate results.

Indoor Models

Statistical models for indoor propagation are more difficult to derive than models for outdoor propagation because of the much wider range of building materials and widely varying configurations of interior spaces. For example, propagation in an open warehouse with metal walls and ceilings would be much different than propagation in a wood-frame house with drywall walls and vinyl siding.

One approach is to use a power-law model with the exponent depending on the type of construction. This can be augmented with a model that includes an attenuation factor for each wall, ceiling or floor that needs to be crossed. The utility of these models for propagation prediction is limited because it is often easier to measure the path loss directly than to figure out the RF properties of the materials used in a particular building.

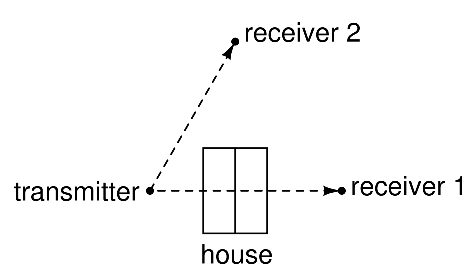
Models for Planning versus Deployment

Statistical models are useful for system-level planning purposes. They help answer questions such as how many cellular base stations will be required to cover an area or the impact of mounting antennas at street level instead of on top of buildings.

However, statistical path-loss models are not typically used for deciding where to place a specific antenna. If we have topological information for an area such as terrain elevations, street widths and building heights then it's possible to estimate the elevation contour between any two locations. From these contours we can get more reliable estimates of the path loss than by using a statistical model.

Propagation planning tools for deployment purposes compute the elevation contour and the corresponding path loss for thousands of paths between a candidate transmitter location and locations within the coverage area. The resulting path loss distribution is used to estimate the fraction of the coverage area that would be adequately covered.

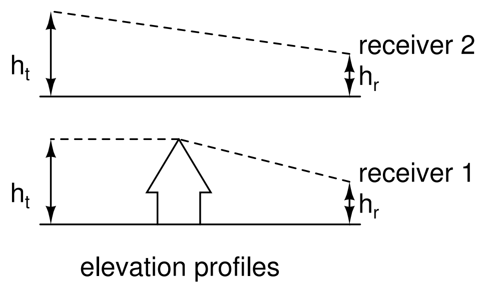
For example in the following location we know the location and height of the transmitter and receivers:



plan view

[1] As given in the COST 231 Final Report, 1998

For this transmiter location we can then compute the elevation profiles for different test points (receivers):



For each of these paths we can estimate the path loss using models similar to the knife-edge diffraction model. By computing the path loss for a large number of locations we can estimate what fraction of the area will have coverage.

Various feasible transmitter locations can be evaluated in the office and the one(s) that gives the best coverage are selected for field ("drive") testing.

Drive Testing and Site Surveys

Propagation predictions always need to be verified. Terrain data is often incomplete and the path loss values generated by the propagation prediction tools are approximations.

Field measurements to validate propagation predictions are typically called "drive testing" for outdoor systems and "site surveys" for indoor systems.

Typically a number of candidate base station site locations will be chosen based on records of spatial call density (for example, more calls placed in malls, fewer in parks), and databases of available base station locations.

Planners then use deployment planning tools (and experience) to identify the candidate locations most likely to provide the required coverage.

Test transmitters and antennas are then set up at candidate sites and technicians drive (or walk) around the coverage area and collect signal quality data along with location data from GPS. The collected data is then used to check the predicted results.

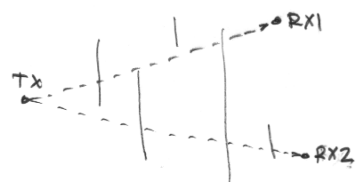
Log-Normal Fading (Shadowing)

Statistical path loss models provide us with an estimate of the mean path loss as a function of distance and other parameters such as frequency and antenna heights. Since these models treat the path loss as a random variable, it is useful to know not just the mean but also the distribution of this random variable. This would allow us to predict how much variability we can expect and compensate for this variability by providing additional margin so as to provide reliable coverage.

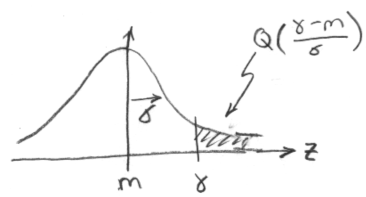
Experimentally it has been found that the path loss in cluttered multipath environments is log-normally distributed. This means that the log of the path loss (for example, the path loss expressed in dB) has a normal distribution. A normal distribution is defined by two parameters: the mean, which is provided by the statistical path loss models and the variance which is typically estimated based on the type of propagation environment (WLAN, rural cellular, etc).

An explanation for the log-normal shadowing distribution is that for any path there are many factors contributing to the overall path loss (combinations of free space loss, diffraction, reflection, transmission, etc). Each of these losses is a random variable. The total loss expressed in dB will be sum of all of these losses (each in dB). The central limit theorem states that the distribution of the overall loss (in dB) will trend to a normal distribution.

Another name for this effect is "log-normal shadowing". This refers to a model where the signal "shadowed" by a variable number of objects and this results in the observed log-normal distribution.



This variation can be modelled as a random variable, X_σ, whose logarithm has a normal (Gaussian) distribution. When X_σ is expressed in dB, is has zero mean and standard deviation σ dB.



The probability that a normal random variable, z, with mean m and standard deviation σ will exceed the value γ is:

$$\Pr[z > \gamma] = Q(\frac{\gamma - m}{\sigma}) = \frac{1}{2}\mathrm{erfc}\left(\frac{\gamma - m}{\sqrt{2}\,\sigma}\right)$$

Exercise 5: A cellular system is designed so that users on the cell edge have an average SNR of 16 dB. The system requires that users have a minimum SNR of 8dB to place a call. The standard deviation of the log-normal fading is 8dB. What fraction of users at the cell edge will be able to place calls?