Multi-Antenna Systems

Introduction

As described earlier in the course, using selection, combining or switching diversity with multiple receive antennas can improve performance in fading channels.

Multiple antennas can also be used to increase the capacity of a system by:

- transmitting on the same frequency simultaneously to multiple users (known as Space Division Multiple Access, SDMA), or
- transmitting multiple data streams in parallel to one user (known as Spatial Multiplexing, or Multiple Input Multiple Output or MIMO), or
- some combination of the two (multi-user MIMO or MU-MIMO).

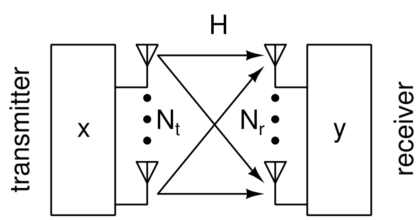
In these types of multi-antenna systems the RF signal on each antenna is converted to/from/ baseband. The transmitter or the receiver implements SDMA or MIMO by combining the baseband signals as described below.

Exercise 1: Would a WiFi system be more likely to use multiple antennas for MIMO or SDMA? How about a cellular system?

Vector Channels

The operation of multi-antenna systems is most easily described using vector and matrix operations.

Consider a system with N_r receive antennas and N_t transmit antennas:



Since the channel is linear[1], the transmitted signals reaching each receive antenna are superimposed (add up). This means we can express the signal received at each antenna as a linear combination of the signals at the different transmit antennas:

$$y = Hx$$

where H is an $N_r \times N_t$ complex matrix representing the loss and phase shift introduced by propagation over the channel, x is an $N_t \times 1$ complex vector representing the amplitude and phase of the transmitted signal and y is an $N_r \times 1$ complex vector representing the signal received by each receive antenna.

Note that the elements of x and y represent the complex values of one (sub-)carrier at one instant of time at one frequency. Also, to keep things simple, we will ignore noise.

Exercise 2: Consider a 2×2 channel where H is $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and x is $[1, -1]$. Find y.

Multi-antenna systems can be used to establish $\min(N_t, N_r)$ parallel channels as long as we can invert the channel matrix H.

Exercise 3: By (up to) what factor could a MIMO system with 3 transmit and 4 receive antennas increase throughput?

MIMO

In the MIMO case the transmit and receive antennas are located on the same device (or base station) and the purpose is to increase capacity.

For example, if we transmit a vector x of $N_t = N_r = N$ data values on each of the N transmit antennas, and then multiply the received signal vector by H^{-1}, the matrix inverse of H, we have:

$$H^{-1}y = H^{-1}Hx = x$$

and we have recovered the transmitted vector, x, the N transmitted data values.

Inverting the effect of the channel requires that we know H, often termed Channel State Information (CSI).

The receiver can estimate CSI if the transmitter transmits known pilot symbols using different transmit antennas.

If CSI is known at the transmitter it's also possible to "precode" the transmit signal x by multiplying it

[1] At least, up until we reach very high power levels where the air ionizes.

by H^{-1} so that the received signal is $y = HH^{-1}x$.

To obtain CSI the transmitter needs to use feedback from the receiver or use TDD and assume that channel is reciprocal and does not change between receive and transmit time slots. Since it's more difficult to obtain accurate CSI at the transmitter, this approach is not as commonly used.

SDMA

In the SDMA case the N_r antennas belong to different users. To transmit to N different users we can precode the transmit signal x, the data values to be transmitted to the N users, by multiplying by H^{-1} so that the received signal is $y = x$.

As explained above, we need CSI at the transmitter for this to work.

A similar approach can be used for receive SDMA.

Implementation

Since the above equations only apply at one frequency and time, the channel must be estimated at all frequencies and updated as the channel state changes. Multi-antenna systems typically use OFDM and preambles or pilot symbols that allow the receiver to periodically estimate CSI at each subcarrier.

The computation of the channel matrix inverse is relatively simple for small N (e.g. 2 or 3) but complexity increases as $\approx O(N^3)$.

Note that this computation must be done for each subcarrier and each frame.

Channel Properties

For the MIMO or SDMA model described above to work, it must be possible to invert the channel matrix. However, we know that singular matrices (where the determinant is zero) are not invertible. For a matrix to be invertible the rows must be linearly independent[2] (not linear combinations of each other).

In general, we can say the following about a matrix:

- it represents a system of N linearly independent equations
- has rank N
- $N \geq$ (typically $=$) number of non-zero eigenvalues

The properties of the channel matrix will depend on the geometry of antenna placements and the scattering environment. If we place $N_t = N_r = N$ antennas in random locations and have a large number of random scattering objects then it's likely that the channel matrix will have rank N and it will be possible to increase the capacity by N.

However, there are situations where this will not be the case. In particular, if two antennas are (nearly) co-located or equidistant from two or more transmit antennas then the rank of the channel matrix will be reduced and it will not be possible to obtain the same degree of parallelism.

Exercise 4: Which channel matrix is more likely to be full-rank (rank N), one for a LOS channel or one for a NLOS channel?

Example

As a simple example consider MIMO transmission of two values in parallel over a 2×2 channel (the same principles apply with more antennas):

```
% the data we want to transmit
x=[1+j;-1+j]
% the channel
h=[1,0.5+2j;3-j,4]
% what we receive on the two antennas:
y=h*x
% the recovered data:
inv(h)*y
```

Capacity

The Shannon capacity of a MIMO system depends on various factors including the channel matrix eigenvalues, the type of fading, and the type of channel state information available at the transmitter and receiver.

There are two special cases with simple results:

- under ideal conditions, N equal-magnitude eigenvalues, the channel capacity of NxN MIMO increases linearly with N,
- if the channel is rank 1 then N antennas increases the SNR by N and capacity goes up approximately as $\log_2(N)$.

[2] Note that this is linear independence, not statistical independence.