# RF Design - Noise

## Noise Temperature

Noise is an unpredictable (random) voltage that is superimposed on a signal. There are various sources of noise. For example, the random motion of electrons in any conductor results in "thermal" noise. These noise sources have a broad spectrum, typically constant over the band of interest.

Because the noise is broad-band we typically measure the noise power spectral density (PSD) and reference it to the thermal noise PSD that would be generated by a resistor at a given temperature. This PSD is $kT$ where $k$ is Boltzmann's constant, and $T$ is the temperature in Kelvin. When passed through a filter with a (brick-wall or integrated) bandwidth $B$, the noise power is:

$$N = kTB \ .$$

Using the equation above we can convert an RF noise power $N$ to a "Noise Temperature," $T$.

## Noise Figure

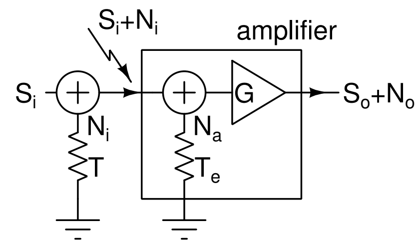
A measure of the noise performance of a device that is useful for calculations involving several devices operating in cascade (series) is the ratio of the input SNR to the output SNR:

$$F = \frac{S_i/N_i}{S_o/N_o} = \frac{S_i/S_o}{N_o/N_i} \ .$$

For example, an attenuator with loss $L$ attenuates the signal so that $S_i/S_o = \frac{1}{L}$ but since the thermal noise level is not affected $N_o = N_i$. Thus the noise figure of an attenuator is equal to $\frac{1}{L}$, the attenuation.

A practical amplifier not only amplifies the input but adds additional noise. We can convert the noise power added by the amplifier to an equivalent increase in the temperature of the input, $T_e$. Thus the apparent noise added at the input is $N_a = kT_eB$.

Consider an amplifier with gain $G$:




The amplifier outputs a signal power $S_o = GS_i$. If the noise input power is $N_i = kTB$ then the noise output power is $N_o = G(N_i + N_a) = Gk(T + T_e)B$. The noise figure is thus:

$$F = \frac{G \cdot k(T+T_e)B}{G \cdot kTB} = \frac{T+T_e}{T} \ .$$

Since $F$ varies with the input noise temperature, a reference input temperature, $T = T_0 = 290$ K is conventionally used when specifying a device's noise figure. $kT_0 = 4 \times 10^{-21}$W/Hz $= -174$ dBm/Hz for $T_0 = 290$ K.

**Exercise 1:** What are the minimum possible values of $T_e$ and $F$?

**Exercise 2:** The datasheet for a low-noise amplifier (LNA) specifies a noise figure of 2 dB. What is the noise temperature $T_e$?

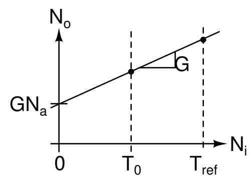
Note that the input noise temperature $T$ varies depending on what is connected to the input of the amplifier. The value $T_0 = 290$ K is typical for an antenna pointed at the ground. But $T$ may be much lower for an antenna pointed into deep space, and much higher for a directional antenna pointed at the sun.

Cryogenically-cooled low-noise amplifiers used for radio astronomy and satellite ground stations can have noise temperatures of a few K. However, conventional low-noise amplifiers (LNAs) have noise temperatures significantly higher than their physical temperature.

The SNR ratio definition for noise figure can be used for other two-port devices such as mixers (ignoring the LO terminal). Although one-port devices such as oscillators generate noise, they do not have noise figures.

**Exercise 3:** An LNA with a noise figure of 0.3 dB receives a signal with an SNR of 6 dB. What is the output SNR?

A device's noise figure can be determined by measuring the output noise powers for two known noise input levels (values of $T$). These noise signals can be obtained from calibrated noise sources. Since the input and amplifier-generated noise signals are independent, their powers will add and it's possible to solve for $N_a$, $T_e$ (and $F$).

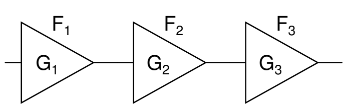



## Cascade Noise Figure

When multiple devices (amplifiers, attenuators, mixers, etc) are connected in series (cascade) both the signal and noise generated by each stage are amplified by subsequent stages. However, while the signal is amplified by each stage, the noise contributed by one stage is only amplified by the succeeding stages. Thus the impact of noise added by later stages has less impact on the overall SNR and thus has less impact on the overall noise figure.

We can use the same output/input SNR-ratio definition of noise figure for the cascaded system as for a single device.

Friis, the Bell Labs researcher who derived the free-space path loss formula, also derived a formula for the noise figure of a cascade of devices. It is possible to show that when several devices (amplifiers, attenuators, mixers, etc) with gains $G_1, G_2, \ldots$ and noise figures $F_1, F_2, \ldots$ (both in linear units) are connected in series (cascade):




the overall (or "system" or "cascade") noise figure is given by:

$$F = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \ldots$$

and the equivalent noise temperature is given by:

$$T_e = T_1 + \frac{T_2}{G_1} + \frac{T_3}{G_1 G_2} + \ldots$$

The significance of these formulas is that the noise figure of the first amplifier has the most impact on the overall system noise figure. Thus the first ("front end") amplifier in a receiver should be chosen to have the lowest possible noise figure and enough gain to reduce the impact of the noise figure of subsequent stages.

**Exercise 4:** A What is the system noise figure of a receiver that consists of a 10 dB amplifier with 3 dB noise figure followed by a mixer with a 6 dB loss and an IF amplifier with a 20dB gain and a noise figure of 10 dB?

## Cascade IP3

When multiple amplifiers are connected in series (cascade) the signal level at the input to the second amplifier is higher than the level at the input to the first stage. We would thus expect that the IP3 of the cascade would be determined primarily by the IP3 of the final amplifier.

It is possible to show that the input IP3 (in linear units) of a cascade of amplifiers with gains $G_1, G_2, \ldots$ and input IP3's $I_1, I_2 \ldots$ is:

$$\frac{1}{IIP3} = \frac{1}{I_1} + \frac{G_1}{I_2} + \frac{G_1 G_2}{I_3} + \ldots$$

When the gains are significant, the IIP3 of the last stage predominates.