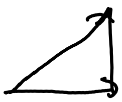
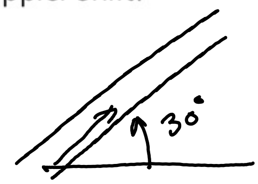
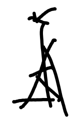
Multipath Fading

Exercise 1: A receiver in a car receives a 1.8 GHz signal while travelling on a road at 50 km/h. The road is at an angle of 30 degrees relative to the direction of arrival of the signal. What is the velocity relative to the direction of arrival of the signal? By how much does the path length change each second (in meters)? In wavelengths? What is the Doppler shift?

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{1.8\times10^9} = m/\text{wavelength}$$







$$50 \cdot \cos 30 \quad km/h.$$

$$V_{m/s} = \frac{50{,}000}{3600} \cdot \cos 30 = 12 \ m/s$$

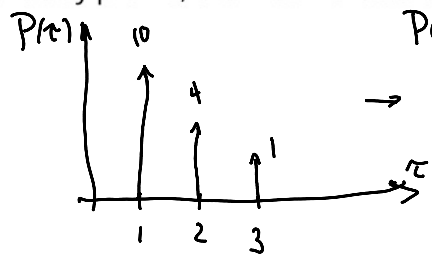
50000/3600 cos30=

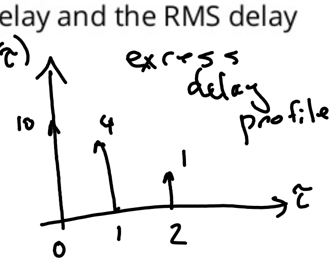
12.02813061

$$V_{wl/s} = \frac{V_{m/s}}{\lambda}$$

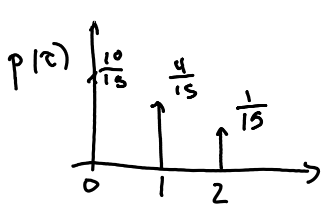
$$f_D = \frac{v}{c} \cdot f_c = \frac{12}{3\times10^8} \cdot 1.8\times10^9 = 4\times10^{-8} \cdot 1.8\times10^9 = 72 \ Hz$$

Exercise 2: A channel has three multipath components with delays of 1, 2 and 3 μs and amplitudes of 10, 6 and 0 dBm respectively. What are the excess delays, the power delay profile, the normalized power delay profile, the mean excess delay and the RMS delay spread?





$$\sum P(\tau) = 10 + 4 + 1 = 15$$

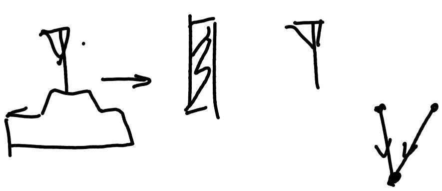


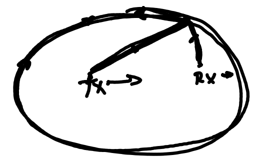
$$\bar{\tau} = \sum p(\tau)\tau = \frac{10}{15}\cdot 0 + \frac{4}{15}\cdot 1 + \frac{1}{15}\cdot 2 = \frac{6}{15}$$

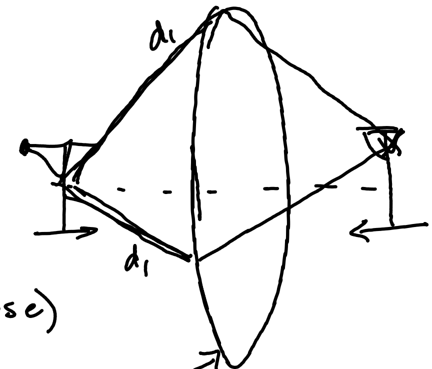
$$\sigma = \sqrt{\sum p(\tau)(\tau-\bar{\tau})^2} = \sqrt{\frac{10}{15}\left(0-\frac{6}{15}\right)^2 + \frac{4}{15}\left(1-\frac{6}{15}\right)^2 + \frac{1}{15}\left(2-\frac{6}{15}\right)^2}$$

$$\approx 0.6$$

Exercise 3: Imagine a receiver traveling in a straight line towards a transmitter but with no LOS path. How could you arrange reflecting objects such that there was no time dispersion (flat fading)? What arrangement would result in no time-varying fading? Neither?







no time dispersion : a single path length (e.g. ellipse)

no time-varying fading: stationary scatterers at one path length

scattering objects on an ellipse

both: same

Exercise 4: What fraction of the time is a Rayleigh-distributed signal 10dB below the mean? 20dB? 30dB? This is a useful result to remember.

"10 dB below the mean" means −10 dB

10 dB : $\frac{V_2}{V_1} = 10^{\frac{-10}{20}}$
0.316

$$\rho = \frac{R}{R_{ms}} = 0.316 \quad (-10\text{ dB})$$

$$P(r \le R) = \int_0^R p(r)\,dr = 1 - e^{-\rho^2}$$

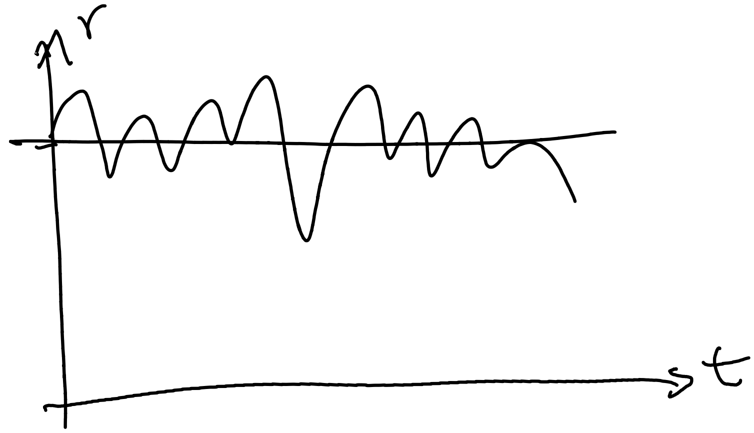
$$= 1 - e^{-\rho^2} = 1 - e^{-(0.316)^2} = 0.095 \approx 0.1$$

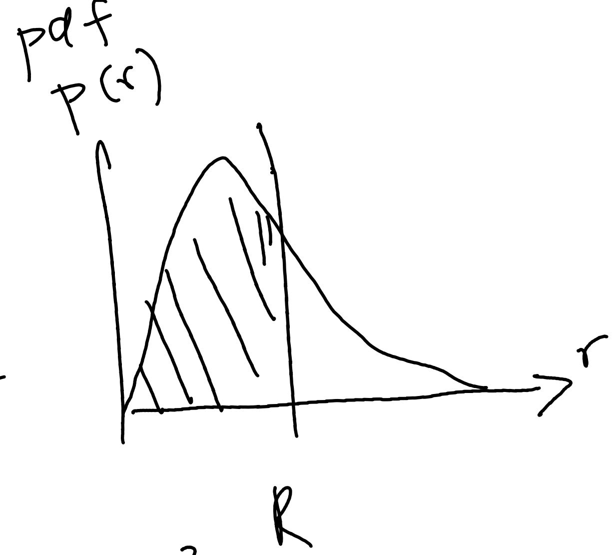
for $\rho = -20\text{dB} = 0.1$

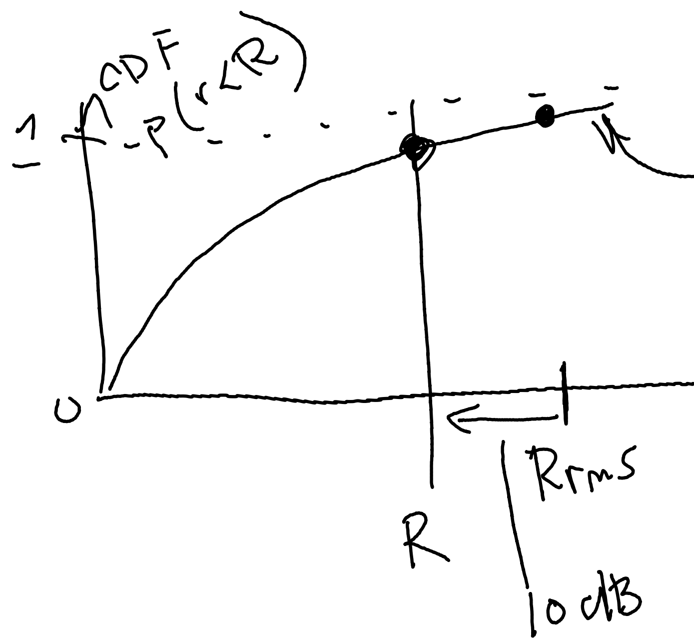
$$P(r \le R) \approx 0.00995 \approx 0.01$$

$$\rho = -30\text{dB} = 0.0316$$

$$P(r < R) \approx 0.001$$







$$1-e^{-\rho^2}$$

$$\rho = \frac{R}{R_{ms.}}$$

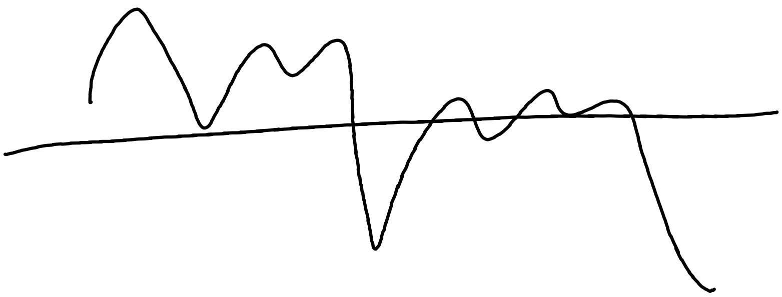
$$R = \rho R_{ms}$$

$$20 \log\left(\frac{R}{R_{ms}}\right) = -10$$

$$\rho = \frac{R}{R_{ms}} = 10^{-\frac{10}{20}} \approx 0.316$$

$$P(r<R) = 1-e^{-(0.316)^2} \approx 0.1$$

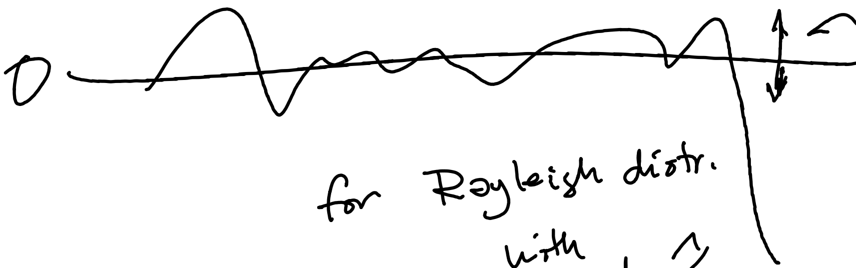
$\bar{x} = 1.25\hat{\sigma}$



first moment

DC

$(\bar{x})^2 =$ "DC power"



$\sigma^2 = \overline{(x - \bar{x})^2}$

= "AC power"

(second central moment).

$\sigma^2 \approx 0.43\hat{\sigma}^2$

for Rayleigh distr.
with
parameter $\hat{\sigma}$

$\overline{x^2}$ (rms power)

"DC+AC" power

$= 2\hat{\sigma}^2$

$\left((1.25)^2 + .43\right)\hat{\sigma}^2$

Exercise 5: How often will the signal drop 10dB below the mean if the carrier frequency is 1.8 GHz and the velocity is 100 km/h? On average, how long will each of these fades last?

$$f_m = \frac{1.8\times10^9 \cdot 100 \frac{km}{hr} \cdot \frac{1}{3600}\, s/hr \cdot 1000 \frac{m}{km}}{3\times10^8}$$

$$= 167 \text{ Hz}$$

$$N_R = \sqrt{2\pi} f_m \rho e^{-\rho^2}$$

$$\rho = \frac{R}{R_{ms}} = 10^{-\frac{10}{20}} = 0.316 = \frac{1}{\sqrt{10}}$$

e fade duration is:

$$N_R = \sqrt{2\pi}\cdot 167 \cdot (0.316)\, e^{-0.316^2}$$

$$= 120 \text{ Hz}$$

$$\bar{\tau} = \frac{e^{\rho^2} - 1}{\rho f_m \sqrt{2\pi}}$$

$$\bar{\tau} = \frac{e^{\frac{1}{10}} - 1}{\frac{1}{\sqrt{10}}\, 167 \cdot \sqrt{2\pi}} = 0.008$$

Exercise 6: What is the product of N_R and $\bar{\tau}$? How does this compare to $P(r \le R)$? Why?

$$N_R \cdot \bar{\tau} = 1 - e^{-\rho^2} = P(r < R)$$