Solutions to Final Exam

Question 1

The Friis equation predicts the received signal level over a line-of-sight path:

$$P_R = P_T G_T G_R \left(\frac{\lambda}{4\pi d}\right)^2$$

In this question $P_T = 100$ mW, $G_T = G_R = 1$, $\lambda = {}^c/_f = {}^{3\times10^8}/_{2.4\times10^9} = 0.125$ m, and $P_R = 10^{-40/10} = 10^{-4}$ mW in one version of the question and $P_R = 10^{-46/10} = \frac{1}{4} \times 10^{-4}$ mW in another.

Solving for the distance from the router, d:

$$d = \sqrt{\frac{P_T}{P_R} \cdot \left(\frac{\lambda}{4\pi}\right)^2} = \sqrt{\frac{100}{10^{-4}} \cdot \left(\frac{0.125}{4\pi}\right)^2} \boxed{\approx 10 \text{ m}}$$

and $\boxed{\approx 20 \text{ m}}$ for $P_R = -46$ dBm.

Question 2

The power-law relationship between distance and path loss in dB is:

$$PL(d) = PL_{dB}(d_0) + 10n \log\left(\frac{d}{d_0}\right)$$

Subtracting the two path losses at the two distances, d_1 and d_2 and simplifying:

$$PL(d_1) - PL(d_2) = 10n\left(\log\left(\frac{d_1}{d_0}\right) - \log\left(\frac{d_2}{d_0}\right)\right)$$

$$= 10n \log\left(\frac{d_1}{d_2}\right)$$

and we can solve for n:

$$n = \frac{PL(d_1) - PL(d_2)}{10 \log\left(\frac{d_1}{d_2}\right)}$$

Since the transmit power is the same in both cases, the difference in path losses must be the same as the difference in received signal powers. In both versions of the question the distance increased from $d_1 = 1.25$ to $d_2 = 2.5$. In both versions of the question the signal power decreased by 9 dB (either from -51 to -60 or from -48 to -57) and so the path loss increased by 9 dB. Solving for n:

$$n = \frac{-9}{10 \log\left(\frac{1.25}{2.5}\right)} \boxed{\approx 3}$$

Question 3

The question asks for the fraction of classrooms with an average signal level that is 1 standard deviation (σ) above the mean. From the graph in the Appendix the probability of being below 1σ above the mean is 84.13% so the probability of being *above* this level is $\approx 1 - 0.84 = \boxed{\approx 16\%}$.

Question 4

If the antennas fade independently then the joint probability is the product of the marginal probabilities so $P(\text{all faded}) = 0.01 = (0.2)^n$ where n is the number of selection-diversity antennas. Solving for n:

$$n = \frac{\log(0.01)}{\log(0.2)} \approx 2.86$$

Since the number of antennas must be an integer, the minimum number of antennas required is $\boxed{3}$.

Question 5

(a) An OFDM symbol with N samples transmitted at a rate f_s has a duration of N/f_s. A guard time of T_g increases this to $N/f_s + T_g$. For $N = 256$, $f_s = 4$ MHz, and $T_g = 5\,\mu$s the symbol period is $256/4 \times 10^6 + 5 \times 10^{-6} = 69\,\mu$s. For $N = 128$ the the symbol period is $128/4 \times 10^6 + 5 \times 10^{-6} = 37\,\mu$s. The corresponding symbol rates are and $\boxed{\approx 14.5\,\text{kHz}}$ and $\boxed{\approx 27\,\text{kHz}}$.

(b) The number of bits are transmitted per (OFDM) symbol is equal to the number of subcarriers used per symbol (given as 200 and 100 in this case) multiplied by the number of bits per used subcarrier ($\log_2(16) = 4$ for 16-QAM), or $200 \times 4 = \boxed{800 \text{ bits/symbol}}$ and $100 \times 4 = \boxed{400 \text{ bits/symbol}}$.

(c) The transmitted bit rate is the symbol rate multiplied by the number of bits per symbol, or $\approx 14.5\,\text{kHz} \times 800 =$ and $\boxed{11.6\,\text{Mbps}}$ $\approx 27\,\text{kHz} \times 400 = \boxed{10.8\,\text{Mbps}}$.

Question 6

There were two versions of this question with different generator matrices:

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

and

$$G = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

We can find the codewords by multiplying the four possible 2-bit messages by the generator matrix to obtain the four codewords. For example, for the first generator matrix $[00]G = [0000]$, $[01]G = [0111]$, $[10]G = [1001]$, and $[11]G = [1110]$.

We can then find the distances between the codewords:

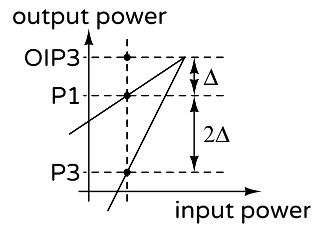
	0000	0111	1001	1110
0000	0	3	2	3
0111		0	3	2
1001			0	3
1110				0

from which we can see that the minimum distance is $\boxed{2}$.

Since the other generator matrix can be obtained from the first by permuting the order of the columns, the only effect is permuting the order of the bits in the codewords and so it has the same minimum distance.

Question 7

- The relationship between the power of the third-order intermodulation products and the output power is shown in the following diagram:



In the diagram the difference between the amplifier's OIP3 (either 30 or 40 dBm in the two versions of this question) and the output power ($P_1 = 10$ or 20 dBm) is $\Delta = 20$ dB in both cases. The power of the third-order intermodulation product components, P_3, is 2Δ below the output power which is either $10 - 40 = \boxed{-30\text{ dBm}}$ or $20 - 40 = \boxed{-20\text{ dBm}}$.

- The frequencies of the third-order intermodulation components for two-tone inputs at frequencies ω_1 and ω_2 are $2\omega_1 - \omega_2$ and $2\omega_2 - \omega_1$. In one version of the question they are at: $2 \cdot 2406 - 2407 = \boxed{2405\text{ MHz}}$ and $2 \cdot 2407 - 2406 = \boxed{2408\text{ MHz}}$. In the other version they are at $2 \cdot 5102 - 5104 = \boxed{5100\text{ MHz}}$ and $2 \cdot 5104 - 5102 = \boxed{5106\text{ MHz}}$.

Question 8

The LNA has a gain of $G_1 = 6$ dB (4 in linear units) and a noise figure of $F_1 = 3$ dB (2). The mixer has a loss of 6 dB (4) so its noise figure is $F_2 = 4$.

The overall (cascade) noise figure is given by:

$$F = F_1 + \frac{F_2 - 1}{G_1} = 2 + \frac{4-1}{4} = 2.75$$

which is $\boxed{\approx 4.4\text{ dB}}$.