Multipath Fading

Introduction

Wireless communication systems are often designed to operate with propagation conditions that include multiple non line-of-sight (NLOS) paths.

Over distances of a few wavelengths, typically less than a few meters, the average path loss will not change very much.

However, over distances of a few wavelengths the *phase* of each component will typically change a few times over the range 0 to 2π. When these multiple components combine at the receiving antenna, the result is a signal whose amplitude and phase that can change greatly over distances on the order of a few wavelengths.

Since wireless systems often include moving transmitters and receivers (or interacting objects), the received signal amplitude and phase will vary with time.

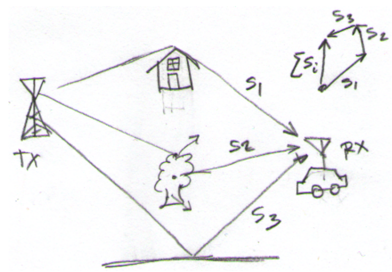
Recovering information from a signal that varies upredictably in amplitude and phase over time is the key challenge in designing wireless communication systems.

This type of fading is known by various names:

- *small-scale fading*: because it changes over distances on the order of a wavelength
- *fast fading*: because it changes faster than other fading effects such as distance or shadowing by buildings
- *"multipath" fading*: because this sort of fading only happens when there are multiple propagation paths
- *"Rayleigh"* fading: because this is the most common resulting statistical distribution of the envelope (the magnitude of the complex envelope representation of a signal)

Physical Model of the Multipath Channel

Regardless of the propagation mechanism, we can model the received signal as the sum of multiple delayed versions of the transmitted signal.



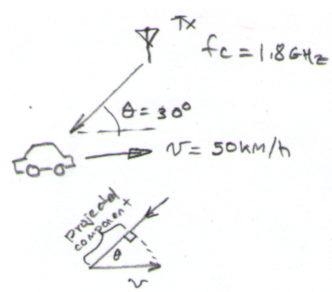
Doppler Shifts

In some cases the path lengths may be changing because the transmitter, receiver and/or the objects are moving relative to each other. This causes the received phase to change at a constant rate which is equivalent to a frequency shift. This frequency shift is called a "Doppler" shift.

The Doppler shift is given by:

$$f_D = \frac{v}{\lambda} = \frac{v}{c} f_c$$

where c is velocity of light, v is the rate of change of path length, and f_c is the frequency of the signal with wavelength λ.



Exercise 1: A receiver in a car receives a 1.8 GHz signal while travelling on a road at 50 km/h. The road is at an angle of 30 degrees relative to the direction of arrival of the signal. What is the velocity relative to the direction of arrival of the signal? By how much does the path length change each second (in meters)? In wavelengths? What is the Doppler shift?

Dispersive Fading

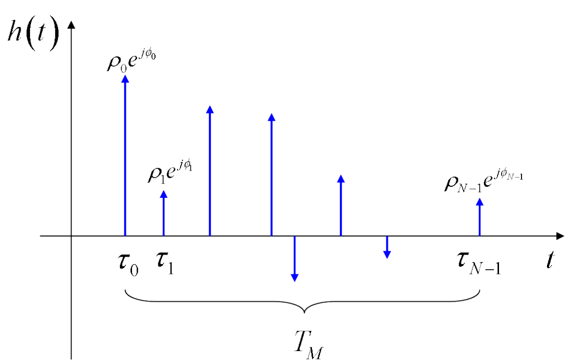
If the delays differ by a significant fraction of the symbol period then the signal will be distorted by inter-symbol interference (ISI). Otherwise, the signal level may be affected but the waveform will not be significantly distorted.

Another way to look at this distinction is to consider the situation in the frequency domain. Since the phase shift is the product of frequency and delay, the signal level resulting from multipath propagation will also be vary with both frequency and delay. If the bandwidth of the signal is significantly smaller than the variation with frequency of the path loss then the signal will not be distorted, otherwise it will.

The situation where the fading does not affect the signal's spectrum is called frequency-flat (or just "flat") fading; the other situation is called "frequency-selective" fading. The channel that exhibits the latter type of fading is also a "dispersive" channel because the signal is dispersed (spread) in time.

Measures of Dispersion

A multipath channel has an impulse response consisting of one impulse for each multipath component. Each has the corresponding delay and amplitude (example from Wikipedia):



We can quantify the time dispersion of the signal using various metrics. Two popular ones are the first and second central moments of the "power delay profile," $P(\tau) = h^2(\tau)$, the square of the impulse response. These are called the "mean excess delay":

$$\bar{\tau} = \sum p(\tau)\tau$$

and the "RMS delay spread" of the channel:

$$\sigma = \sqrt{\sum p(\tau)(\tau - \bar{\tau})^2}$$

where $p(\tau)$ is the normalized power delay profile:

$$p(\tau) = \frac{P(\tau)}{\sum P(\tau)} \, .$$

The minimum observed delay has no effect on dispersion and should be subtracted out when computing $\bar{\tau}$.

Exercise 2: A channel has three multipath components with delays of 1, 2 and 3 μs and amplitudes of 10, 6 and 0 dBm respectively. What are the excess delays, the power delay profile, the normalized power delay profile, the mean excess delay and the RMS delay spread?

Similarly, the frequency response of the channel will, in general, not have a well-defined shape and we can define a quantity called the "coherence bandwidth" which can be used to quantify the frequency selectivity of the channel. The coherence bandwidth is the frequency range over which the fading at two frequencies are well-correlated.

It is also possible to define a "coherence time", related to the Doppler rate, during which the fading on the channel is well-correlated.

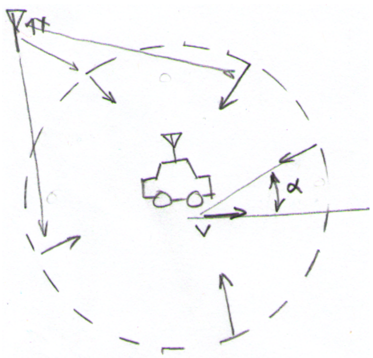
Note that the frequency selectivity (dispersiveness) of the channel and how the channel changes with time (the fading in time) are independent of each other. The former is a function of the path lengths (and thus delays) while the latter is a function of velocity of objects. Thus it is possible to have a dispersive channel that does not experience fading and flat-fading (non-dispersive) channel. The nature of the channel and its effect on the signal thus depends on the propagation environment and motion through it.

Exercise 3: Imagine a receiver traveling in a straight line towards a transmitter but with no LOS path. How could you arrange reflecting objects such that there was no time dispersion (flat fading)? What arrangement would result in no time-varying fading? Neither?

Flat-Fading Model

Clarke developed a simple model whose predictions agree reasonably well with the statistics of flat-fading NLOS channels.

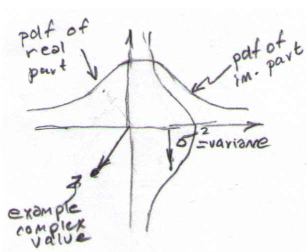
The model consists of a large number of signal paths arriving from directions that are uniformly distributed in a circle around a moving receiver. Each path has equal loss (amplitude) but random phase (0 to 2π). Since the sources are equidistant, all delays are equal and the channel is flat (not dispersive).



Envelope Distribution

The probability distribution of the amplitude of the signal can be obtained by decomposing the (complex) vector sum of the different paths into real and imaginary components. According to the Central Limit Theorem the real and imaginary components will then be normally distributed since each is the sum of a large number of independent random variables (r.v.'s).

The probability density function (pdf) of the magnitude of a complex r.v. whose real and imaginary components are normally distributed is Rayleigh distributed. The following diagram tries to show the pdfs of the real and imaginary components and a sample value drawn from the distribution:

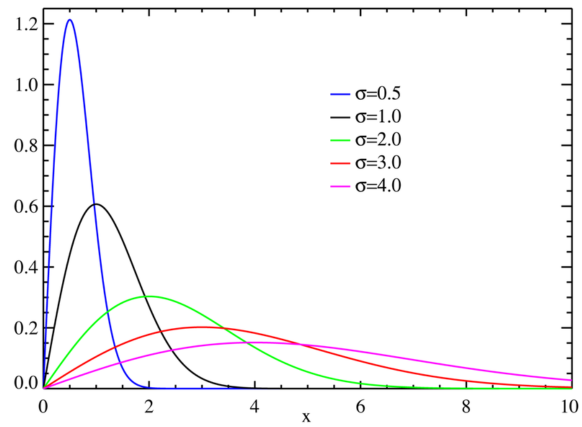


The Rayleigh pdf has the form:

$$p(r) = \begin{cases} \frac{r}{\sigma^2} \exp\left(-\frac{r^2}{2\sigma^2}\right) & r \geq 0 \\ 0 & r < 0 \end{cases}$$

The Rayleigh distribution has only one parameter, σ^2, which is the variance of each component (real or imaginary) of the signal.

The following plot of the Rayleigh pdf is from the Wikipedia article:



The corresponding cumulative distribution of a Rayleigh random variable is:

$$P(r \leq R) = \int_0^R p(r)\, dr = 1 - e^{-\rho^2}$$

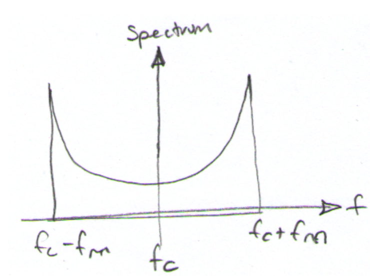
where $\rho = R/R_{rms}$ where $R_{rms} = \sigma\sqrt{2}$ is the power (second moment or "AC+DC" power) of the signal.

Exercise 4: What fraction of the time is a Rayleigh-distributed signal 10dB below the mean? 20dB? 30dB? This is a useful result to remember.

The mean (first moment or "DC" power) of the Rayleigh distribution is $\bar{r} = \sigma\sqrt{\frac{\pi}{2}} \approx 1.25\sigma$ and its variance (second central moment or "AC" power) can be found to be $\sigma^2\left(2 - \frac{\pi}{2}\right) \approx 0.43\sigma^2$.

Doppler Spectrum

The signal will be spread in frequency due to the Doppler shifts of the (infinite number of) components. Each component will have a Doppler shift proportional to the cosine of the angle α relative to the direction of motion (see diagram above). Assuming an omnidirectional antenna the Doppler spectrum has a "bathtub" shape extending over the range $f_c \pm f_m$ where f_m is the maximum Doppler shift ($f_m = f_c v/c$):



Level Crossing Rate and Mean Fade Duration

From the power spectrum it is possible to derive two useful time-domain statistics. The level crossing rate is the rate at which the received signal level crosses a threshold ρ in one direction. The level crossing rate is:

$$N_R = \sqrt{2\pi} f_m \rho e^{-\rho^2}$$

and the average fade duration is:

$$\overline{\tau} = \frac{e^{\rho^2} - 1}{\rho f_m \sqrt{2\pi}}$$

Exercise 5: How often will the signal drop 10dB below the mean if the carrier frequency is 1.8 GHz and the velocity is 100 km/h? On average, how long will each of these fades last?

Ricean Fading

A common situation is that there are both LOS and NLOS components. In this case the received signal is the sum of a constant component and a Rayleigh-distributed component. The resulting probability distribution is Ricean. The ratio of the powers of the direct and Rayleigh components is given by the parameter called the "Ricean K factor":

$$K(dB) = 10 \log \frac{A^2}{2\sigma^2}$$

where A is the amplitude of the direct (LOS) component.