Solutions to Final Exam

Question 1

(a) The free-space path loss in dB is given by

$$-20\log\left(\frac{c}{4\pi \cdot f \cdot d}\right).$$

There were two versions of this question ($d = 300$, $f = 6$ GHz and $d = 600$, $f = 3$ GHz) both giving a path loss of $L_{\text{path}} \approx$ $\boxed{97.5\,\text{dB}}$.

(b) The Friis equation where all quantities are in dB or dBm is:

$$P_R = P_T + G_T + G_R - L_{\text{path}}\,.$$

There were two versions of this question: $G_T = G_R = 6$ dB, $P_T = 100\,\text{mW} = 20\,\text{dBm}$ giving $P_R \approx$ $\boxed{\text{-65.5 dBm}}$ and $G_T = G_R = 9$ dB, $P_T = 200\,\text{mW} = 23\,\text{dBm}$ giving $P_R \approx$ $\boxed{\text{-56.5 dBm}}$.

Question 2

There were two values of the maximum Doppler shift, $f_m = \frac{v}{c} f_c$:

- $v = 20$ m/s, $f_c = 1$ GHz $\implies f_m = 66.7$ Hz
- $v = 30$ m/s, $f_c = 2$ GHz $\implies f_m = 200$ Hz

A fading threshold of ρ_{dB} dB relative to the mean is $\rho = 10^{\rho_{\text{dB}}/20}$ (or $\rho^2 = 10^{\rho_{\text{dB}}/10}$) in linear units. There were two fading thresholds:

- $\rho_{\text{dB}} = -13\,\text{dB} \implies \rho = 0.224$
- $\rho_{\text{dB}} = -16\,\text{dB} \implies \rho = 0.158$

(a) The level crossing rate is given by:

$$N_R = \sqrt{2\pi} f_m \rho e^{-\rho^2}$$

(b) The average duration is given by:

$$\overline{\tau} = \frac{e^{\rho^2} - 1}{\rho f_m \sqrt{2\pi}}$$

The following table gives the level crossing rate and fade duration for the four versions of the question:

v (m/s)	ρ (dB)	N_R (Hz)	τ (s)
20	−13	35.6	1374×10^{-6}
20	−16	25.8	960×10^{-6}
30	−13	106.7	457×10^{-6}
30	−16	77.5	320×10^{-6}

Question 3

The problem specifies a sampling rate $f_s = 4$ MHz, an FFT size of $N = 256$ and an occupied bandwidth of 3.125 MHz.

(a) If N_{used} subcarriers are used, the signal bandwidth will be $B = \frac{N_{\text{used}}}{N} f_s$ (or $B = \frac{N_{\text{used}}+1}{N} f_s$ depending on the definition of bandwidth). This results in $N_{\text{used}} = \frac{NB}{f_s}[-1] = \frac{256 \cdot 3.125}{4}[-1] =$ $\boxed{200 \text{ or } 199}$.

(b) The OFDM symbol duration is $T = \frac{N}{f_s} = 256/4 = 64\,\mu\text{s}$

(i) With a 16 or 8 μs cyclic extension the symbol duration becomes $\boxed{80 \times 10^{-6}\,\text{s} \text{ or } 72 \times 10^{-6}\,\text{s}}$.

(ii) The symbol rate is the inverse of this: $\boxed{12\,500\,\text{Hz or } 13\,890\,\text{Hz}}$.

(c) The overall bit rate will be the product of the number of subcarriers used, the OFDM symbol rate and the number of bits per subcarrier. There were four possible answers:

N_{used}	f_{symbol} (Hz)	$\frac{\text{bits}}{\text{subcarrier}}$	bit rate (bps)
200	12.5×10^3	2	5×10^6
199	12.5×10^3	2	4.975×10^6
200	13.89×10^3	4	11.110×10^6
199	13.89×10^3	4	11.056×10^6

Question 4

There were two versions of the question with the generator matrices:

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix} \text{ or } \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

for an $(n, k) = (5, 3)$ FEC code.

(a) A systematic code has a generator matrix of the form:

$$G = \left[I_k \,|\, P \right]$$

and the parity check matrix can be obtained as:

$$H = \left[P^T \,|\, I_{n-k} \right]$$

Which in this case are:

$$\boxed{H = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix} \text{ or } \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \end{bmatrix}}$$

(b) The transmitted codeword is obtained as the matrix multiplication (using addition modulo-2) of the $1 \times k$ data vector $1, 1, 0$ with the $k \times n$ generator matrix G. The computations for the two generator matrices is as follows:

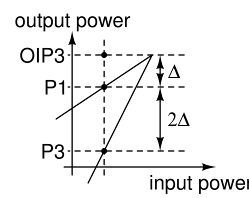
$$1 \cdot 1 \oplus 1 \cdot 0 \oplus 0 \cdot 0 = 1 \oplus 0 \oplus 0 = 1$$
$$1 \cdot 0 \oplus 1 \cdot 1 \oplus 0 \cdot 0 = 0 \oplus 1 \oplus 0 = 1$$
$$1 \cdot 0 \oplus 1 \cdot 0 \oplus 0 \cdot 1 = 0 \oplus 0 \oplus 0 = 0$$
$$1 \cdot 1 \oplus 1 \cdot 0 \oplus 0 \cdot 1 = 1 \oplus 0 \oplus 0 = 1$$
$$1 \cdot 0 \oplus 1 \cdot 1 \oplus 0 \cdot 0 = 0 \oplus 1 \oplus 0 = 1$$

$$1 \cdot 1 \oplus 1 \cdot 0 \oplus 0 \cdot 0 = 1 \oplus 0 \oplus 0 = 1$$
$$1 \cdot 0 \oplus 1 \cdot 1 \oplus 0 \cdot 0 = 0 \oplus 1 \oplus 0 = 1$$
$$1 \cdot 0 \oplus 1 \cdot 0 \oplus 0 \cdot 1 = 0 \oplus 0 \oplus 0 = 0$$
$$1 \cdot 0 \oplus 1 \cdot 1 \oplus 0 \cdot 1 = 0 \oplus 1 \oplus 0 = 1$$
$$1 \cdot 1 \oplus 1 \cdot 1 \oplus 0 \cdot 0 = 1 \oplus 1 \oplus 0 = 0$$

Thus the codewords transmitted are $\boxed{1, 1, 0, 1, 1}$ or $\boxed{1, 1, 0, 1, 0}$.

Question 5

When measured in dB, the output third-order intermodulation products are 2Δ lower than the desired signals and the desired signal is Δ below the output IP3:



In this question $2\Delta = 40$ dB and the desired signal level is 20 dBm. The output IP3 is $20 + \Delta = 20 + 40/2 = 40$ dBm. 40 dBm is $1 \times 10^{40/10} = 10^4$ mW = $\boxed{10\text{ W}}$.

Question 6

The noise figure of a cascade of two amplifiers is given by:

$$F = F_1 + \frac{F_2 - 1}{G_1}.$$

In this question we are given $F = 3.5\text{ dB} = 10^{3.5/10} = 2.239$, $F_1 = 2\text{ dB} = 1.585$ and in two versions of the question $F_2 = 6\text{ dB} = 4$ or $F_2 = 9\text{ dB} = 8$. Solving for G_1:

$$G_1 = \frac{F_2 - 1}{F - F_1} = \frac{(4 \text{ or } 8) - 1}{2.239 - 1.585} = 4.6 \text{ or } 10.7$$

which are $10 \log(4.6) = \boxed{6.6\text{ dB}}$ or $10 \log(10.7) = \boxed{10.3\text{ dB}}$.