Modulation - Part 2

After this lecture you should be able to:

- *for the common digital phase/frequency modulations methods (BPSK, DBPSK, QPSK, GMSK):*
  - *give the equation for the signal*
  - *give signal-space basis functions and constellation*
  - *list some implementation advantages and disadvantages*
  - *compute the error rate in an AWGN channel*
- *compute performance of these modulation techniques over a flat, slow-fading channel*

5.5.2 Raised-Cosine Filter

- used to smooth out the baseband modulating signal to limit the bandwidth without causing ISI (it meets the Nyquist criteria for no ISI).
- transfer function is given in Equation 1 and the impulse response in Equation 2:
- for $\alpha = 0$ this is a rectangular "brick-wall" filter from 0 to $\frac{1}{2T_s}$ (the minimum bandwidth)
- for $\alpha = 1$ this is a half-cycle of raised cosine extending from 0 to $\frac{1}{T_s}$ (double the minimum bandwidth)
- root raised-cosine (RRC) filter has a transfer function that is the square root of a RC filter so that we can place half of the filter at the transmitter and half at the receiver

5.5.3 Gaussian Filter

- has Gaussian transfer function and impulse responses:

$$H_G(f) = \exp(-\alpha^2 f^2)$$

where

$$\alpha = \frac{\sqrt{\ln 2}}{\sqrt{2}B} = \frac{0.5887}{B}$$

and B is the -3dB bandwidth of the filter

- impulse response also Gaussian:

$$h_G(t) = \frac{\sqrt{\pi}}{\alpha} \exp\left(-\frac{\pi^2}{\alpha^2} t^2\right)$$

- does not meet Nyquist criteria, so will introduce ISI
- symmetrical, non-causal

Probability of Error Analysis 5.6

- using orthonormal basis functions we can express each of the M possible transmitted symbols as M sets of N coordinates along N dimensions
- the orthonormal basis functions are typically sinusoids:

$$\phi_j(t) = \sqrt{\frac{2}{T_b}} \cos(2\pi f_c t + \theta) \qquad 0 \le t \le T_b$$

where f_c, T_b and θ are chosen so as to make the sinusoids orthogonal.

Exercise: What values of θ make these basis functions orthogonal? What values of f_c make these basis functions orthogonal?

- for example, if we choose $N = 2$ one choice would be to use $\theta = 0$ and $\theta = \pi/2$.
- then to each of the M symbols we would assign a pair of numbers

$$H_{RC}(f) = \begin{cases} 1 & 0 \leq |f| \leq \frac{1-\alpha}{2T_s} \\ \frac{1}{2}\left[1+\cos\left(\frac{\pi[(2T_s|f|)-1+\alpha]}{2\alpha}\right)\right] & \frac{1-\alpha}{2T_s} \leq |f| \leq \frac{1+\alpha}{2T_s} \\ 0 & 0 \leq |f| > \frac{1+\alpha}{2T_s} \end{cases} \tag{1}$$

$$h_{RC}(t) = \left(\frac{\sin(\pi t/T_s)}{\pi t}\right)\left(\frac{\cos(\pi\alpha t/T_s)}{1-(4\alpha t/(2T_s))^2}\right) \tag{2}$$

- the receiver converts the received waveform into N coordinates using N matched filters and then chooses the symbol that lies nearest the received coordinate

- each matched filter multiplies the received signal by one of the basis functions and integrates over the symbol duration, T_b

- a bound on the probability of symbol (not bit) error given symbol s_i is transmitted is:

$$P(e|s_i) \leq \sum_{\substack{j=1 \\ j\neq i}}^{M} Q\left(\frac{d_{ij}}{\sqrt{2N_0}}\right)$$

where $Q(\cdot)$ is "Markum's Q function" and N_0 is the power spectral density of the noise at the input to the matched filters.

- for equally-likely symbols we can just average the individual symbol error probabilities:

$$P(e) = \frac{1}{M}\sum_{i=1}^{M} P(e|s_i)$$

5.7, 5.8 Linear and Constant-Envelope Modulation

- typically linear modulation [1] multiplies the carrier with the modulating signal:

$$s(t) = Re[m(t)e^{j2\pi f_c t}]$$

- the resulting envelope (magnitude) may not be fixed, in which case any subsequent amplifier must be linear (e.g. Class A, AB, etc)

[1] Linearity requires that superposition apply: if $m(t) = m_1(t)+m_2(t)$ then $s(t) = s_1(t)+s_2(t)$

- a phase-modulated signal:

$$s(t) = Re[e^{j2\pi f_c t + m(t)}]$$

is an example of non-linear modulation

- this signal *does* have a constant envelope (magnitude)

- signals with constant envelope do not require linear power amplifiers

- a non-linear RF power generators (e.g. a class C "amplifier") is usually more power-efficient and less expensive than a linear amplifier and less expensive than "linearized" PAs

- note that linear modulation can also be constant-envelope (if the envelope of m(t) is fixed)

Exercise: Give an example of an $m(t)$ that results in constant-envelope linear modulation.

- many constant-envelope variants of phase modulation have been developed

- they differ in the way the data is filtered before modulating the phase of the carrier and in their modulation index (ratio of frequency deviation to bit rate)

- the pre-modulation filter is a compromise between spectrum main-lobe width (bit rate), spectrum roll-off (adjacent channel interference), BER performance and transmitter/receiver complexity

BPSK 5.7.1

- Binary Phase Shift Keying

- if we have one sinusoidal basis function ($N = 1$) and two symbols $M = 2$, we can choose:

$$s(t) = m(t)\sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_c t)$$

where $m(t)$ consists of pulses of duration T_b and amplitude ± 1.

Exercise: Is this a type of linear modulation? Is it a constant-envelope signal?

- The power spectrum of the envelope (without filtering) is:

$$P(f) = 2E_b\left(\frac{sin(\pi f T_b)}{\pi f T_b}\right)^2$$

- this spectrum is very wide, and typically a low-pass filter (e.g. RC) is used to limit the bandwidth

Exercise: Does filtering $m(t)$ change the answers to the previous exercise?

- a BPSK receiver has to generate a local signal ($\phi(t)$) of the correct frequency and phase
- this is difficult on a multipath channel because there are many time-varying paths being summed
- for mobile applications, typically a pilot signal or pilot symbols are transmitted to allow the receiver to estimate and remove the net phase shift applied by the channel
- the probability of error is:

$$P_e = \frac{1}{2}Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

5.7.2 DBPSK

- Differential Binary Phase Shift Keying
- when the phase is changing quickly or a simpler receiver is necessary, we can encode the data as a phase difference from symbol to symbol. In the case of $N = 2$ we can represent a 0 bit by no change of phase and a 1 bit by a phase change of 180 degrees.
- the advantage of differential encoding is that we don't need an absolute phase reference to demodulate the signal – we just compare the phase difference from one bit to the next (e.g. by using a delay and a complex multiplier).
- the probability of error of DBPSK in an AWGN channel is worse than BPSK:

$$P_e = \frac{1}{2}\exp\left(-\frac{E_b}{N_0}\right)$$

however, this assumes the BPSK has perfect synchronization and that the DBPSK receiver only looks at one previous symbol. In practice both systems can be made to perform about the same

QPSK 5.7.3

- Quadrature Phase Shift Keying
- in this case $N = 2$ and $M = 4$
- the basis functions are two carriers in quadrature:

$$s(t) = \sqrt{\frac{2E_s}{T_s}}\cos[2\pi f_c t + i\frac{\pi}{2}]$$

for $i = 0, 1, 2, 3$

Exercise: Draw the constellation diagram for QPSK.

- error performance is same as BPSK:

$$P_e = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

- can transmit 2 bits per symbol without increasing the bandwidth or error rate!
- can also use differential coding (DQPSK)

O-QPSK and $\pi/4$-QPSK 5.7.5 – 5.7.8

- although all QPSK points have the same amplitude the trajectory from one point to another may not be along a constant radius if I and Q changing simultaneously and instantaneously

- large changes in amplitude are thus possible which cause the spectrum to broaden
- two alternatives are offset QPSK and $\pi/4$ QPSK which take slightly different approaches to trying to constrain the trajectory to keep it along the circle

5.8 Constant-Envelope Modulation

- constant envelope means don't need linear amplifiers, resulting in higher transmitter efficiency
- pre-modulation filtering is sufficient to reduce adjacent-channel interference levels to adequate levels (-60 dB) so no additional post-modulator filter is required
- can use inexpensive discriminator-based receiver
- many, many variants (BFSK, MSK, GTFM, ...)
- first-null spectrum is wider than BPSK or QPSK, but steeper roll-off

5.8.3 GMSK

- Gaussian Frequency Shift Keying
- uses Gaussian filtering of baseband pulse
- filter introduces ISI but produces well-defined spectrum
- Gaussian filter impulse response and transfer function given above
- parameter BT is the product of bit period and 3dB filter bandwidth
- lower BT provides more filtering, narrow bandwidth but more ISI and higher residual error rate floor
- error rate:

$$P(e) = Q\left(\sqrt{\frac{2\gamma E_b}{N_0}}\right)$$

  where γ is a parameter determined by BT ranging from 0.68 for $BT = 0.25$ to 0.85 for $BT = \infty$ (MSK).

- transmitter is simply a filter with Gaussian impulse response and an FM modulator
- receiver can be coherent (synchronous) or non-coherent (a discriminator)
- widely used (GSM, CDPD)

M-ary Modulation 5.9

- with $N = 2$ we can create constellations with more than $M = 4$ points and so transmit more than 2 bits/symbol (need $M = 2^n$ points to transmit n bits/symbol)
- the points can be on a circle about the origin (M-ary PSK) or on a rectangular grid (M-ary QAM)
- common on point-to-point links, not as widely used in fading channels
- these higher-level modulations are more sensitive to channel disturbances since the points are closer together and require more signal processing for accurate synchronization and equalization but are being used in newer systems because of increased spectral efficiency

  Exercise: Draw 8-PSK and 16-QAM constellations.

Coded Modulation –

- also known as "trellis-coded modulation" (TCM)
- in larger (e.g. 8-PSK) constellations where points are not equidistant some symbol errors (e.g. between adjacent points) are much more likely than others
- by increasing the number of constellation points and adding redundancy ("coding") it's possible to significantly reduce the error rate

Performance of Modulation Schemes in Fading Multipath Channels 5.11

- will differ from static error rates
- often bursty, need more than BER to characterize (e.g. burst lengths)

Slow, Flat Fading

5.11.1

- if fading is "slow" (relative to symbol period) then the signal is approximately fixed during one symbol period:

$$r(t) = \alpha(t)e^{-\theta(t)}s(t) + n(t)$$

where $\alpha(t)$ is the instantaneous signal amplitude

- we can find the average error probability (P_e) by averaging over the distribution of the signal SNR:

$$P_e = \int_0^\infty P_e(X)p(X)dX$$

where $X = \alpha(t)^2 E_b/N_0$ and the known BER performance in noise, ($P_e(X)$):

- if α is Rayleigh, α^2 has a chi-squared distribution with two degrees of freedom:

$$p(X) = \frac{1}{\Gamma}\exp\left(-\frac{X}{\Gamma}\right)\ X \geq 0$$

where $\Gamma = \frac{E_b}{N_0}\overline{\alpha^2}$ is the mean SNR

- for BPSK:

$$P_e = \frac{1}{2}\left[1 - \sqrt{\frac{\Gamma}{1+\Gamma}}\right]$$

- for DBPSK:

$$P_e = \frac{1}{2(1+\Gamma)}$$

- for GMSK:

$$P_e \approx \frac{1}{4\delta\Gamma}$$

where δ depends on BT

- these results are much worse than the AWGN case (the error rate drops off linearly rather than exponentially with increasing SNR)
- this is because error events are almost all caused by deep fade events, so BER is dominated by pdf of the fading, not the noise
- only reducing the probability of deep fades can improve BER (diversity, coding)

Frequency-Selective Channels 5.11.2

- channel delay spread causes ISI which causes errors
- receiver may need to measure channel impulse response and use an equalizer
- time-varying Doppler spread causes random FM noise which creates an error floor (limits maximum SNR)
- BER results typically derived by simulation