Solutions to Assignment 8 - IIR Filter Design

Question 1

(a) Substituting $s = j\omega$ in the analog filter's transfer function:

$$H(j\omega) = \frac{1}{-\omega^2 + j\omega + 1}$$

the -3 dB frequency will be when

$$|H(j\omega)|^2 = \frac{1}{(1-\omega^2)^2 + \omega^2} = 1/2$$

which are the (real) roots of:

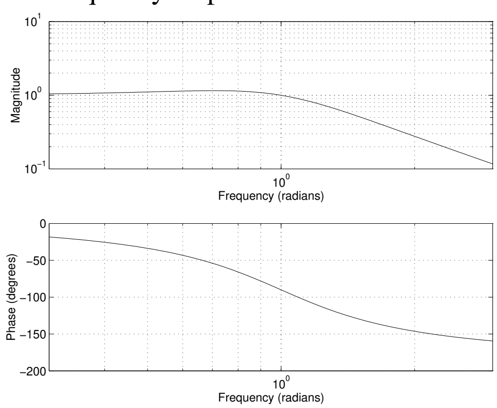
$$\omega^4 - \omega^2 - 1$$

Using the Matlab command `roots([1 0 -1 0 -1]);` the -3dB frequency is $\omega \approx \pm 1.27$ radians/s or 0.2 Hz.

To check the result we can plot the filter's analog amplitude response using the Matlab commands:

```
>> freqs([1],[1 1 1],[0.3:0.01:3]);
>> print -deps sol8.q1a.eps;
```

and the frequency response looks like:



(b) The frequency scale of the digital filter will be warped due to the bilinear z-transform. For a sampling frequency of 10 Hz (T=0.1 s) the corresponding -3 dB frequency of the analog filter will be:

$$\Omega = \frac{2}{0.1} \tan\left(1.27 \frac{0.1}{2}\right) = 1.27$$

radians/s.

In this case the two frequencies are approximately the same since the -3 dB frequency is small relative to the sampling rate. If they had been different we would have had to scale 's' before finding the corresponding z-transform.

To find the z-transform of the desired filter we substitute $s = \frac{2}{0.1}\frac{z-1}{z+1}$ into the original equation. We can do this using Matlab's symbolic math function `compose()`:

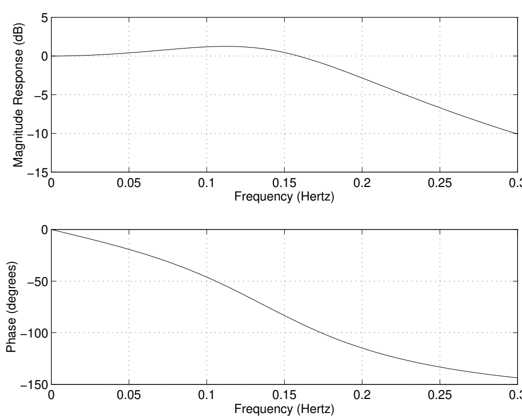
```
>> f='1/(s^2+s+1)';
>> g='20*(z-1)/(z+1)';
>> h=compose(f,g,'s','z');
>> simplify(h)
```

which produces the result:

$$\frac{(z+1)^2}{421\,z^2 - 798\,z + 381}$$

We can check the result by plotting the digital filter's response:

```
>> b=[1 2 1];
>> a=[421 -798 381];
>> freqz(b,a,[0:0.005:0.3],10);
>> print -deps sol8.q1b.eps
```



Question 2

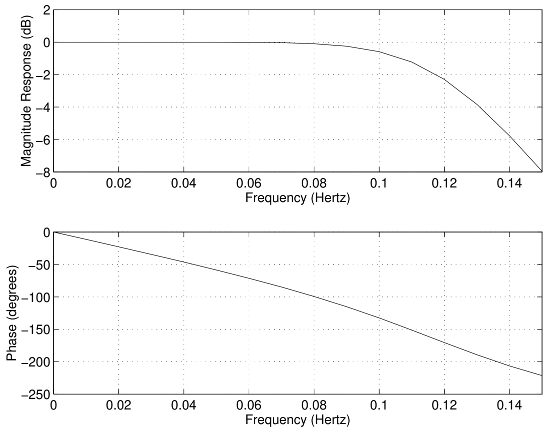
The following Matlab commands design a 4-th order Butterworth LP filter with -3 dB frequency of $F_s/8$

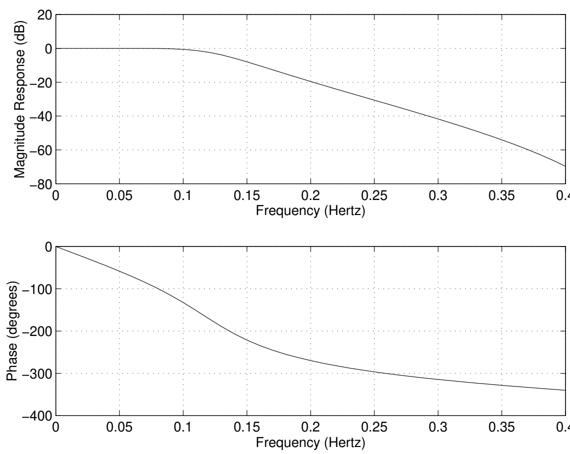
and plot the amplitude and phase response of the filter over the passband and up to about the Nyquist frequency.

```
>> [b a]=butter(4,1/4);
```

```
>> freqz(b,a,[0:0.01:0.15],1);
>> print -deps sol8.q2a.eps
>> freqz(b,a,[0:0.01:0.4],1);
>> print -deps sol8.q2b.eps
```

The plots are as follows:





Question 3

The following commands design an 8-th order Elliptic bandpass filter with 0.5 dB maximum passband ripple, 60 dB minimum stopband attenuation and a passband from 0.5 to 22 kHz assuming a sampling rate of 50 kHz. Note that for bandpass and bandstop filters the first argument is half of the final filter order (the lecture notes don't take this into account).

```
>> [b a]=ellip(4,0.5,60,[0.5/25 22/25]);
```

The following commands plot from DC to $F_s/2$ to verify the stopband and from 0.5 to 22 kHz to verify the passband.

```
>> freqz(b,a,[0:0.1:25],50);
>> print -deps sol8.q3.eps
>> freqz(b,a,[0.5:0.1:22],50);
>> print -deps sol8.q3b.eps
```

The amplitude and phase response of the filter is shown in the following plots: