ELEC 466 : DIGITAL SIGNAL PROCESSING
1995/96 WINTER SESSION TERM 2

Solutions to Assignment 7 - FIR Filter Design

Question 1

(a)

The amplitude response of an ideal linear-phase low-pass filter with a cutoff frequency equal to 0.2 F_s is:

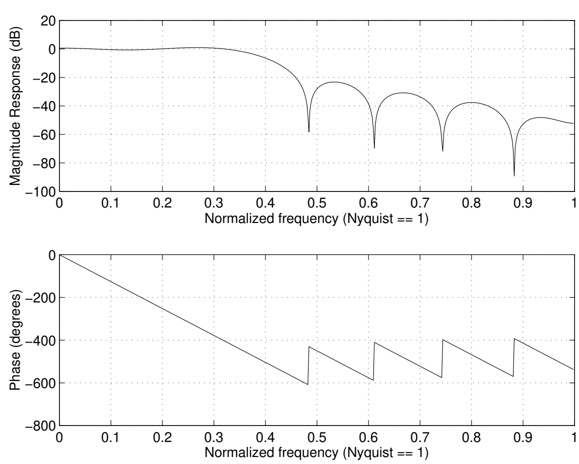
$$H(f) = \begin{cases} 1 & \text{if } |f| < 0.2F_s \\ 0 & \text{otherwise} \end{cases}$$

The impulse response is

$$h(t) = 0.4F_s \text{sinc}(0.4F_s t)$$

The values of the filter coefficients of an FIR filter of length 15 for a direct form filter implementation are obtained by sampling the ideal impulse response at $t = n/F_s$ for $n = -7, -6, \ldots, 7$. The values of the resulting frequency and phase response (with the axes scaled by $1/F_s$) can be plotted using Matlab's `freqz()` function:

```
>> h=0.4*sinc(0.4*[-7:7]);
>> freqz(h,[1]);
>> print -deps sol7.q1a.eps
```

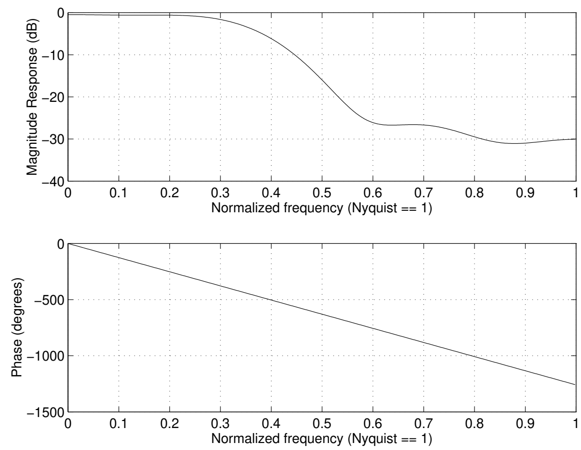


Changing F_s will scale the amplitude and frequency and axes.

(b)

We can multiply the coefficients with a triangular window and re-plot the frequency response:

```
>> freqz(h'.*triang(15),[1]);
>> print -deps sol7.q1b.eps
```



You could also have used the similar `bartlett()` window which is zero at the endpoints.

Question 2

(a) Low-Pass Filter

Since the minimum stopband attenuation is 50 dB the Hamming window is the window that will produce the shortest filter that meets the specifications. The transition width is $5-4 = 2$ kHz, or $0.1F_s$. For a Hamming window the width of the transition region is approximately 3 times the inverse of the impulse response duration. The approximate number of filter coefficients needed is thus $3/0.1 = 30$.

Since this is a low-pass filter, the impulse response is (from Table 6.2):

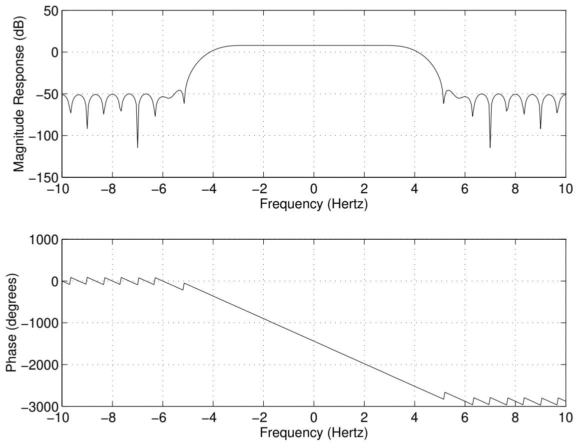
$$2f_c \text{sinc}(2nf_c)$$

for $n = -15, \ldots, 15$ (31 samples total) and where f_c should be the half-way point between the passband and stopband (4 kHz) and normalized to $4/20 = 0.2$. The impulse response can be computed as:

```
>> h=hamming(31)'.*(0.4*sinc(0.4*[-15:1:15]));
```

```
>> freqz(h,[1],[-10:.05:10],20);
>> print -deps sol7.q2a.eps
```

The resulting frequency response is:



(b) Band-Pass Filter

For this filter the minimum stopband attenuation is only 20 dB so we can use a rectangular window. The normalized cutoff frequencies are $f_1 = 4/20 = 0.2$ and $f_2 = 6/20 = 0.3$. The impulse response is given by:

$$h(n) = 2f_2\text{sinc}(2nf_2) - 2f_1\text{sinc}(2nf_1)$$

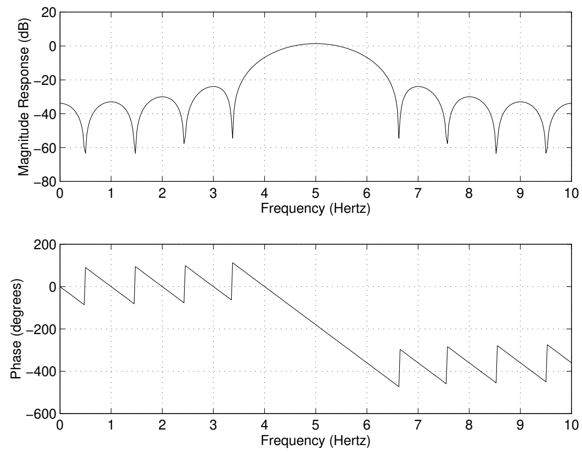
$$= 0.6sinc(0.6n) - 0.4sinc(0.4n)$$

The normalized transition region width is $1/20 = 0.05$ so the window width should be approximately $1/0.05 = 20$ samples.

The Matlab commands to compute the impulse response are:

```
>> n=[-10:10];
>> h=0.6*sinc(0.6*n)-0.4*sinc(0.4*n);
>> freqz(h,[1],[0:0.025:10],20);
>> print -deps sol7.q2b.eps
```

The resulting frequency response is:



Question 3

(a) Low-Pass Filter

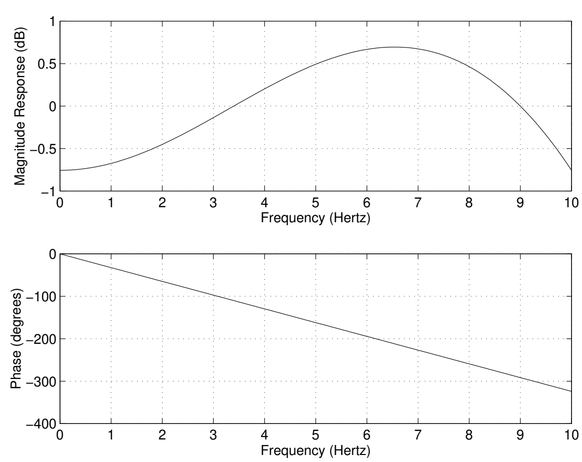
The transition region extends from 10 to 20 kHz, the mean signal levels will be 1 in the passband and 0 in the stopband, the passband ripple will be $10^{0.2/20} - 10^{-0.2/20} = 1.023 - 0.977 = 0.046$, and the stopband ripple will be $2 \times 10^{-60/20} = 0.002$.

```
>> F=[10 20];
>> M=[1 0];
>> DEV=[0.046 0.002];
>> Fs=100;
>> [N Fo Mo W]=remezord(F,M,DEV,Fs)
```

The resulting filter has N = 18, Fo = { 0, 0.2000, 0.4000, 1.0000 }, Mo = { 1, 1, 0, 0 }, and W = { 1, 23 }.

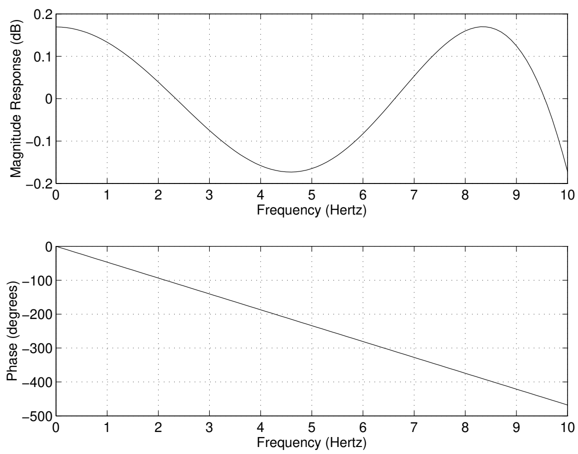
We check the passband:

```
>> B=remez(N,Fo,Mo,W);
>> freqz(B,[1],[0:0.1:10],100);
>> print -deps sol7.q3a1.eps
```



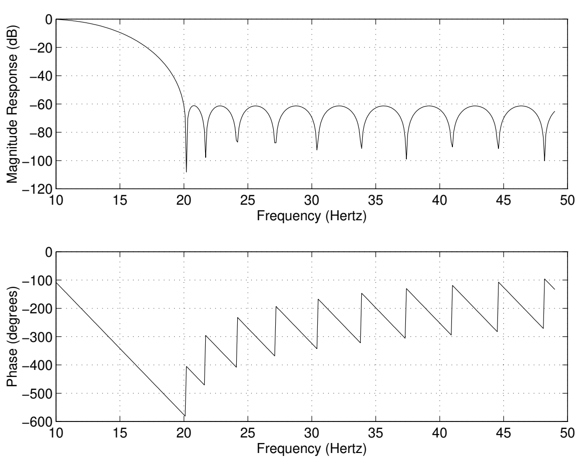
and find that it does not meet the specifications. By increasing N by 8 we find the filter now meets the specifications in the passband:

```
>> B=remez(N+8,Fo,Mo,W);
>> freqz(B,[1],[0:0.1:10],100);
>> print -deps sol7.q3a2.eps
```



and the stopband:

```
>> freqz(B,[1],[10:0.1:49],100);
>> print -deps sol7.q3a3.eps
```



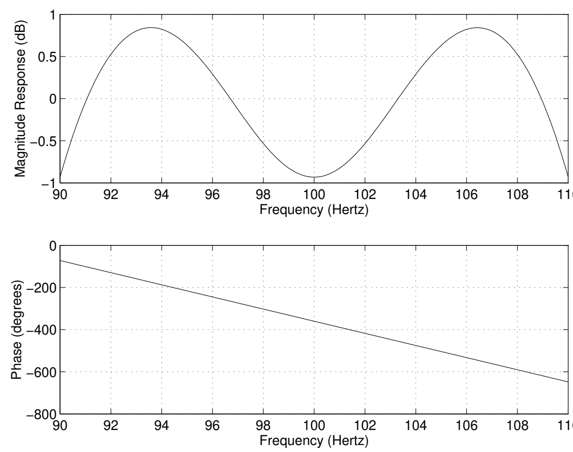
(b) Band-Pass Filter

In this design there are four stopbands and one passband resulting in four transition regions: from 20 to 30 kHz, from 80 to 90, from 110 to 120 and from 170 to 180. As usual the mean signal levels will be 1 in the passband and 0 in the stopband. The peak-to-peak passband ripple will be $10^{1/20} - 10^{-1/20} = 1.122 - 0.891 = 0.231$, and the two stopband ripples will be $2 \times 10^{-30/20} = 0.0632$ and $2 \times 10^{-50/20} = 0.00632$.

```
>> F=[20 30 80 90 110 120 170 180];
>> M=[0 0 1 0 0];
>> DEV=[0.0632 0.00632 0.231 0.00632 0.0632];
>> Fs=400;
>> [N Fo Mo W]=remezord(F,M,DEV,Fs)
```

The resulting filter has N = 64, Fo = { 0, 0.1000, 0.1500, 0.4000, 0.4500, 0.5500, 0.6000, 0.8500, 0.9000, 1.0000 }, Mo = { 0, 0, 0, 0, 1, 1, 0, 0, 0, 0 }, and W = { 3.6551, 36.5506, 1.0000, 36.5506, 3.6551 }.
we design the filter and check the passband:

```
>> B=remez(N,Fo,Mo,W);
>> freqz(B,[1],[90:0.1:110],400);
>> print -deps sol7.q3b1.eps
```



which meets the ± 1 dB ripple specifications. The stopbands also meet the specifications:

```
>> f=[0:0.5:199];
>> h=freqz(B,[1],f,400);
>> axis([0 200 -60 -20]);
>> plot(f,20*log10(abs(h)));
>> print -deps sol7.q3b2.eps
```