Solutions to Assignment 3

Question 1

(a) A median filter is not a linear system. Consider the signals $a(n) = \delta(0)$ and $b(n) = \delta(1)$. The responses of the median filter to the signals applied individually will be 0, but the response to $a(n) + b(n)$ will be $b(n) = \delta(1) + \delta(2)$ which is not equal to the sum of the two individual responses.

(b) The output of a median filter, $y(n)$, for an input $x(n)$ is the median of the values $\{x(n - 2N), x(n - 2N + 1), \ldots, x(n)\}$. The output, $y'(n)$, of a median filter to a delayed input $x'(n) = x(n - m)$ will be the median of the samples $\{x(n - 2N - m), x(n - 2N + 1 - m), \ldots, x(n - m)\}$ which is the output of the median filter for the input $x(n)$ delayed by m. Therefore a median filter is time-invariant.

(c) The impulse response is the response of the system to an input of $\delta(0)$. The output of a median filter at $n = 0$ to an input $\delta(0)$ is the median of the values $\{0, 0, 1\}$ which is 0. the output at $n = 1$ is the median of $\{0, 1, 0\}$ which is also 0. Similarly all other outputs will be zero so the impulse response is 0.

Question 2

Let $y(n) = a^n x(n)$. From the definition of the z-transform

$$Y(z) = \sum_{n=-\infty}^{\infty} a^n x(n) z^{-n}$$

$$Y(z) = \sum_{n=-\infty}^{\infty} x(n) \left(\frac{z}{a}\right)^{-n}$$

which is the z-transform of $x(n)$ evaluated at $\frac{z}{a}$, i.e. $X(z/a)$. Multiplying a sequence by a^n therefore moves the locations of points in the z-transform of $x(n)$ (e.g. poles and zeroes) towards or away from the origin.

Question 3

(a) The z-transform of the sequence $\{0.5, 1.5, 2.5\}$ is $0.5 + 1.5z^{-1} + 2.5z^{-2}$.

(b) By definition the transfer function is the z-transform of the impulse response. The z-transform of $\{0, 1, 3, 5\}$ is $z^{-1} + 3z^{-2} + 5z^{-3}$.

Question 4

(a) The transfer function is already in factored form so we can find the poles and zeroes by inspection. There system has one pole at $z = 0.05$ and two poles at $z = 1.55$ and $z = 0.55j$.

(b) The following commands generate two 41-element vectors of numbers over the range -2 to 2 in increments of 0.1, and use the meshdom() command to generate two 41x41 element matrices in which each element contains the x or y co-ordinate replicated along the row or column.

```
>> x=[-2:0.1:2];
>> y=x;
[a b]=meshdom(x,y);
```

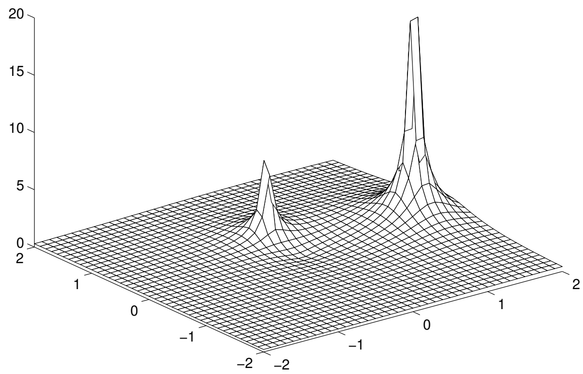
The following command creates a matrix containing complex values whose real and imaginary parts are the x, and y coordinate at each point.

```
z=a+j*b;
```

The following commands compute the magnitude of the transfer function at each point in the matrix 'z' and plot it as a surface:

```
f=abs((z-0.05)./((z-1.55).*(z+0.55*j)));
mesh(x,y,f);
```

The plot looks as follows:



and the locations of the zero at (approximately) the origin and the poles at zeros at (approximately) $z = 1.5 + j0$ and $z = 0 + j0.5$ are clearly visible.

Question 5

(a) From the signal flow graph we can obtain the following difference equation:

$$y(n) = 3y(n-1) - 0.5x(n) - 0.5x(n-1) + x(n-2))$$

or

$$y(n) - 3y(n-1) = -0.5x(n) - 0.5x(n-1) + x(n-2)$$

The z-transform is:

$$Y(z)(1 - 3z^{-1}) = X(z)(-0.5 - 0.5z^{-1} + z^{-2})$$

and the transfer function is:

$$\begin{aligned} H(z) &= \frac{Y(z)}{X(z)} \\ &= \frac{(-0.5-0.5z^{-1}+z^{-2})}{(1-3z^{-1})} \\ &= \frac{(z^2+z-2)}{(-2z^2+6z)} \\ &= \frac{(z-1)(z+2)}{-2z(z-3)} \end{aligned}$$

An alternative solution involves realizing that this filter is in the direct form described in the lecture, obtaining the values of the coefficients a_i and b_i from the signal flow graph ($a_0 = -0.5, a_1 = -0.5, a_2 = 1, -b_1 = 3$) and writing out the transfer function directly:

$$H(z) = \frac{\sum_{k=0}^{N} a_k z^{-k}}{1 + \sum_{k=1}^{M} b_k z^{-k}}$$

$$H(z) = \frac{-0.5 - 0.5z^{-1} + z^{-2}}{1 - 3z^{-1}}$$

(b) This is an IIR filter because it has feedback elements. It is not an FIR filter because it is an IIR filter (the two are mutually exclusive).

(c) The following Matlab commands were used to create a plot of the magnitude of the frequency response of the system above in dB over the range $-3 < \omega < 3$ in steps of 0.1 radian (the warning message is because the transfer function has a zero at $z = e^{j0} = 1$):

```
>> w=[-3:0.1:3];
>> ew=exp(j*w);
>> y=(ew.^2+ew-2)./(-2*ew.^2+6*ew);
>> ydb=20*log10(abs(y));

Warning: Log of zero
>> plot(w,ydb);
>> print -deps sol3.q4.eps
>> quit
```

The plot looks as follows: