Solutions to Assignment 2

Question 1

Consider a new sequence, $y(n)$, that consists of the values of $x(n)$ in reverse order, $y(n) = x(N-n)$, and its DFT:

$$Y(k) = \sum_{n=0}^{N-1} x(N-n) W_N^{kn}$$

If we substitute $m = N - n$, or $n = N - m$:

$$Y(k) = \sum_{m=N,N-1,\ldots,1} x(m) W_N^{k(N-m)}$$

Since $W_N^{k(N-m)} = W_N^{kN} W_N^{-km} = W_N^{-km}$ and re-arranging the order of the sum (which has no effect on the result) we get

$$Y(k) = \sum_{m=1}^{N} x(m) W_N^{-km}$$

Since W_N and $x(n)$ are periodic with period N, $W_N^N = W_N^0$ and $x(N) = x(0)$,

$$Y(k) = \sum_{m=0}^{N-1} x(m) W_N^{-km}$$

which is N times the inverse DFT of $x(n)$. Thus we can use an FFT processor to compute the inverse DFT by reversing the order of the inputs and scaling the result by $1/N$.

Question 2

(a) W_4^k ($e^{-j2\pi k/4}$) is a complex number with magnitude 1 and phase $-2\pi k/4$. The following table shows the values of the real and imaginary parts of W_4^k for various values of k.

k	$\angle W_4^k$	$\mathrm{Re}\{W_4^k\}$	$\mathrm{Im}\{W_4^k\}$
0	0	1	0
1	$-\pi/2$	0	-1
2	$-\pi$	-1	0
3	$\pi/2$	0	1
4	0	1	0

(b) Multiplication by a complex carrier is often used to convert band-pass signals to a complex baseband representation and vice-versa. The results above reveal that choosing a carrier frequency of $F_s/4$ results in a simple implementation since it only involves multiplication by zero and ± 1. In the case of a real input signal the implementation is particularly simple since the real and imaginary parts of the product can be obtained by selecting alternate samples and then negating every second sample in each part.

Question 3

We can compute the DFT sums as follows:

For $k=0$				
n	$x(n)$	kn	W_4^{kn}	$x(n)W_4^{-kn}$
0	1	0	1	1
1	1	0	1	1
2	0	0	1	0
3	0	0	1	0
				$\sum = 2$

For $k=1$				
n	$x(n)$	kn	W_4^{kn}	$x(n)W_4^{-kn}$
0	1	0	1	1
1	1	1	$-j$	$-j$
2	0	2	-1	0
3	0	3	j	0
				$\sum = 1-j$

For $k=2$				
n	$x(n)$	kn	W_4^{kn}	$x(n)W_4^{-kn}$
0	1	0	1	1
1	1	2	-1	-1
2	0	4	1	0
3	0	6	-1	0
				$\sum = 0$

For $k=3$				
n	$x(n)$	kn	W_4^{kn}	$x(n)W_4^{-kn}$
0	1	0	1	1
1	1	3	j	j
2	0	6	-1	0
3	0	9	$-j$	0
				$\sum = 1+j$

The DFT of $\{1,1,0,0\}$ is thus $\{2,1-j,0,1+j\}$.

Question 4

The linearity of the DFT implies that scaling the input of the DFT by 2 will scale the output by the same amount. Since we have found that the DFT of $\{1,1,0,0\}$ is $\{2,1-j,0,1+j\}$, then the DFT of $\{2,2,0,0\}$ must be $\{4,2-2j,0,2+2j\}$

Similarly, the sequence $\{0,2,2,0\}$ is the sequence $\{2,2,0,0\}$ with at (circular) shift of 1. Since $x(n-m) \rightleftharpoons X(k)W^{km}$, the DFT of $\{0,2,2,0\}$ is W_N^k times the original sequence, $\{4,-2-2j,0,-2+2j\}$. The following table shows the computation:

k	$X(k)$	W_4^k	$X(k)W_4^k$
0	4	1	4
1	$2-2j$	$-j$	$-2-2j$
2	0	-1	0
3	$2+2j$	j	$-2+2j$

Question 5

The frequencies of the spectral components of the DFT are given by $f = kF_s/N$. For example, a frequency of $-F_s/2$ corresponds to $k=-N/2$. Since the DFT is periodic with period N, the value of the DFT evaluated at a frequency $-k$ is the same as the value at frequency $N-k$. So the results of the 8-point DFT would be plotted from $k=-4$ to $k=4$ corresponding to indices $k=4$ to $k=7$ and then $k=0$ to $k=4$. The following table shows the relationship between k and frequency for $F_s = 4$kHz:

k	$(k+N)_N$	frequency	kHz
-4	4	$-4F_s/8$	-2.0
-3	5	$-3F_s/8$	-1.5
-2	6	$-2F_s/8$	-1.0
-1	7	$-1F_s/8$	-0.5
0	0	$0F_s/8$	0.0
1	1	$1F_s/8$	0.5
2	2	$2F_s/8$	1.0
3	3	$3F_s/8$	1.5
4	4	$4F_s/8$	2.0

Question 6

Figure 1 on the following page shows the signal flow graph for an 8-point decimation-in-time FFT and the values at each node for the input $\{0,1,1,1,0,1,1,1\}$.

Question 7

The sample indices for a 1024-point FFT are represented by binary numbers that are 10 bits wide. We can find the sample index in bit-reversed order by bit-reversing the indices written as binary numbers. The computation of the indices of the first four reordered inputs to the FFT are given in Table 1 on the next page.

Question 8

in-place An algorithm that computes the FFT using the same storage locations as occupied by the input.

radix-2 A DFT of length 2 is a radix-2 DFT. The term also applies to an FFT computed by using radix-2 DFTs as the basic computational step.

decimation in time The process of decomposing a DFT into the sum of two (or more) DFTs each computed over half (or fewer) of the original samples.

reordered input The ordering of the time-domain input samples to allow in-place computation of

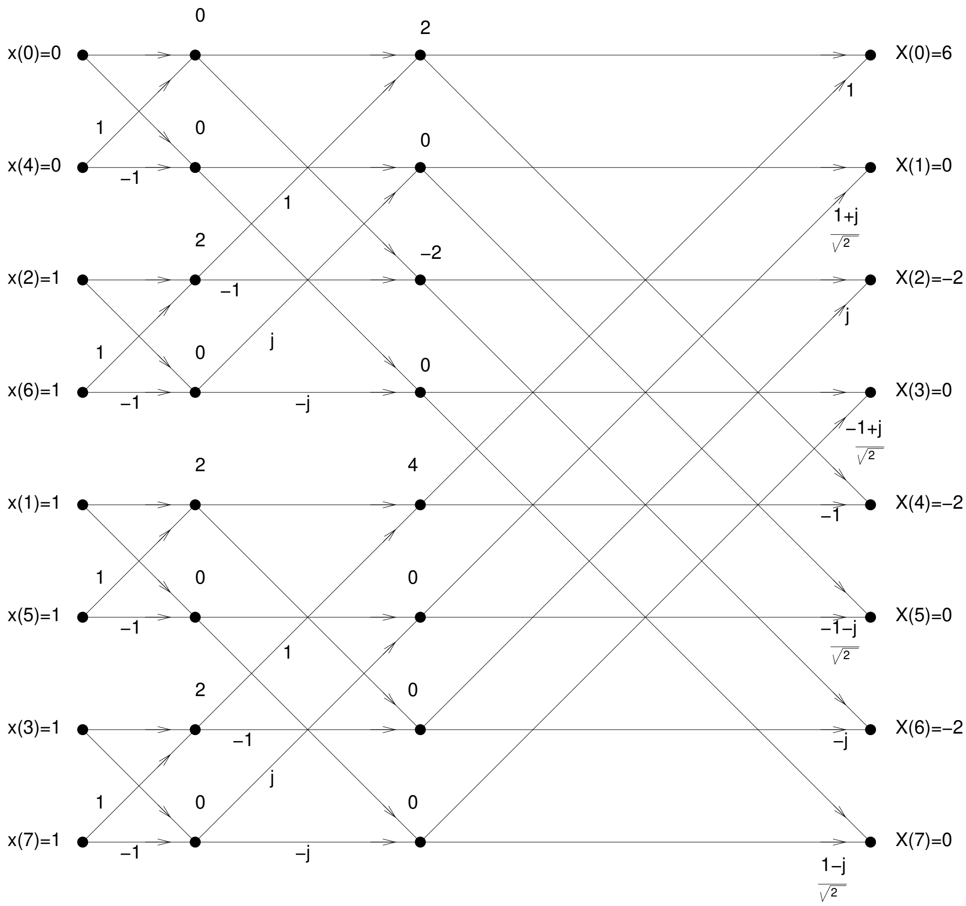

Figure 1: Decimation in Time Radix-2 FFT.

input location	index in binary	reversed binary	required sample index
0	00000 00000	00000 00000	0
1	00000 00001	10000 00000	512
2	00000 00010	01000 00000	256
3	00000 00011	11000 00000	768

Table 1: First four sample indices in bit reversed order for a 1024-point FFT.

a decimation-in-time FFT. This is done by rearranging the input samples in bit-reversed index order.

butterfly The signal flow graph for a 2-point DFT.

twiddle factor A power of W_N used in scaling the result of one stage of the FFT in order to compute the next.