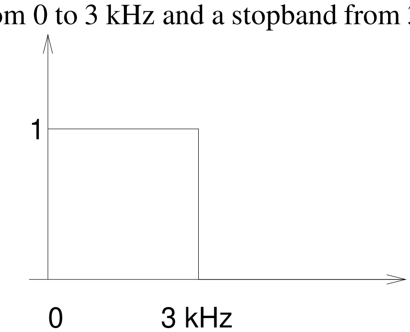
Final Exam Review Solutions

Revised Version.

FIR Design - Window Method

(a) The ideal filter will have a passband (gain=1) from 0 to 3 kHz and a stopband from 3 kHz up.



(b) From table 6.2 in the text, the impulse response of the ideal filter is:

$$h(n) = 2f_c sinc(2nf_c)$$

where $sinc(x) = sin(\pi x)/(\pi x)$.

(c) For the stopband attenuation to be at least 40 dB down we would have to window the ideal impulse response with a Hanning window (-44 dB stopband).

(d) For a Hanning window the ratio of the impulse response duration to transition region width is 3.1. The normalized transition region width is 1 kHz / 10 kHz = 0.1 so the number of filter coefficients required is $3.1/0.1 = 31$. The duration in microseconds is $31/10^3 = 3100$ microseconds.

(e) The filter is a linear-phase filter since the impulse response is symmetrical (even symmetry about zero in this case). To implement this filter the impulse response would be delayed by 16 samples so that it became causal. The magnitude of the transfer function would be unaffected.

The bilinear z-transform

(a) The frequency response is obtained by evaluating $H(j\omega)$. The magnitude will be $1/\sqrt{2}$ when $s = j\omega = j/a$. So the -3 dB frequency of the filter is at $\omega = 1/a$ radian/second.

(b) Warping the frequency response of a digital filter with a –3 dB frequency of 5 Hz for a system with a sampling rate of 10 Hz is not possible since the warping operation would have to warp the frequency response from 5 Hz to infinity into the single frequency of 5 Hz.

Such a low-pass filter would not make much sense anyways since a filter with real coefficients has a frequency response that is symmetrical about half of the sampling rate.

To complete the example we will assume that the –3 dB frequency is 5 radians/second. In this case the analog filter's –3 dB frequency should be:

$$\Omega = \frac{2}{T}\tan\left(\omega\frac{T}{2}\right)$$

and using $T = 1/10$, and $\omega = 5$, we find the $--$ 3dB frequency of the analog filter should be $\Omega = 20\tan(5/20) = 5.1$ radians/second.

Thus the value of a should be $\Omega = 1/a = 5.1$ or $a = 0.196$.

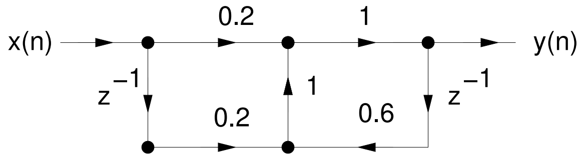
(c) By substituting $s = 20\frac{z-1}{z+1}$ into the s-domain transfer function we obtain the z-transform of the desired filter:

$$H(z) = \frac{z+1}{3.92(z-1)+z+1}$$

$$= \frac{z+1}{5z-3}$$

$$= \frac{0.2z^{-1}+0.2}{-0.6z^{-1}+1}$$

and the signal flow graph is:



Finite Precision Filter Implementation

(a) Multiplying by z^{-2}/z^{-2} the transfer function becomes:

$$H(z) = \frac{z^{-2} + 2z^{-1} + 1}{-3z^{-2} + 2z^{-1} + 1}$$

From $H(z) = \frac{Y(z)}{X(z)}$ we can obtain:

$$Y(z)(-3z^{-2} + 2z^{-1} + 1)$$
$$= X(z)(z^{-2} + 2z^{-1} + 1)$$

or the difference equation:

$$-3y(n-2) + 2y(n-1) + y(n)$$

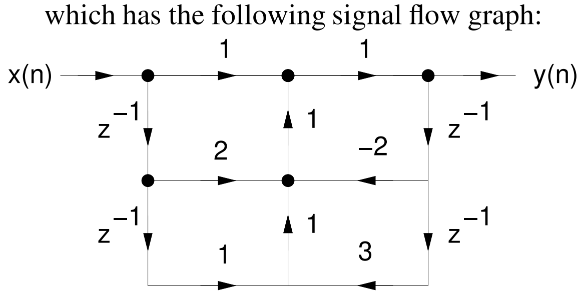
$$= x(n-2) + 2x(n-1) + x(n)$$

or

$$y(n) = x(n-2) + 2x(n-1) + x(n)$$

$$+3y(n-2) - 2y(n-1)$$

which has the following signal flow graph:



(b) To obtain a parallel realization we need to rewrite the second-order transfer function as a sum of two first-order functions. This requires that the transfer function be expanded as a sum of partial fractions. Dividing the numerator by denominator we get:

$$H(z) = 1 + \frac{4}{z^2 + 2z - 3}$$

The second term can be expanded into partial fractions:

$$\frac{4}{z^2 + 2z - 3} = \frac{A}{z-1} + \frac{B}{z+3}$$

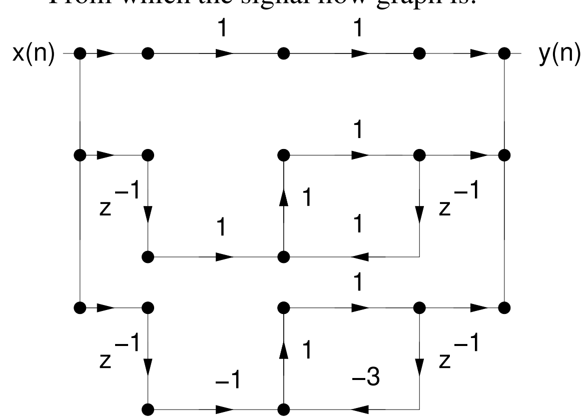
Solving for A and B by substituting $z = -3$ and $z = 1$, we find $A = 1$, and $B = -1$, thus

$$H(z) = 1 + \frac{1}{z-1} + \frac{-1}{z+3}$$

or

$$H(z) = 1 + \frac{z^{-1}}{-z^{-1} + 1} + \frac{-z^{-1}}{3z^{-1} + 1}$$

From which the signal flow graph is:



Decimation and Interpolation

(a) Sampling causes the spectrum of the signal to be replicated about multiples of the sampling frequency. Thus there will be components at $\pm 8n$ kHz ± 0.5 kHz and $\pm 8n$ kHz ± 2 kHz resulting in components at frequencies of . . .,-10, -8.5, -7.5, -6, -2, -0.5, 0.5, 2, 6, 7.5, 8.5, 10, . . . kHz.

Reducing the sampling rate by a factor of 4 corresponds to re-sampling the signal at a frequency of to 2 kHz, and the spectrum is again replicated about $\pm 2n$ kHz. The components between -2 and 2 kHz will be at: -1.5, 0, -2, -0.5, 0.5, 2, 0, and 1.5. The level of the component at 0 Hz will depend on the phase of the original signal.

(b) By inserting two zeros between each sample we effectively increase the sampling rate to 300 kHz but the spectrum of this sampled signal is still symmetrical about the original sampling rate of 100 kHz. Thus the spectrum will consist of 60 kHz wide flat regions about zero, ± 100, ± 200, and ± 300, kHz. The amplitudes of these components will been reduced by a factor of 3 from the original signal due to the addition of the zeros.

Spectral Estimation

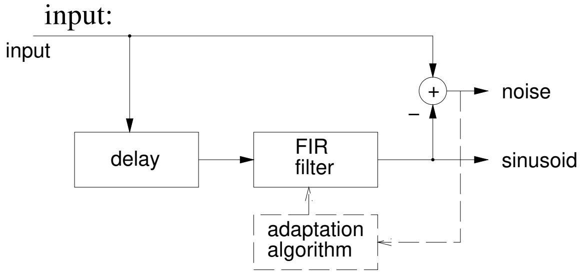
The first zero of the Fourier Transform of a rectangular window function will be at the inverse of the window width $(1/T)$.

To obtain a resolution (using this definition) of 100 Hz the window width must be $1/100 = 10$ ms. For a signal sampled at 10 kHz this corresponds to $10000 \times 0.01 = 100$ samples.

To average 7 periodograms with a 50% overlap, we would need a total of 400 samples (4 full sets of DFT).

Adaptive Filtering

(a) As described in the lecture notes, an adaptive filter can be used to cancel a periodic signal by feeding the filter with a delayed version of the input:



(b) To use the simple bound given in the lecture notes we need to know the variance of the input to the FIR filter. In this case the input is the sum of noise and a sinusoid which are uncorrelated so the powers (variances) will add. The power of a sinusoid of amplitude A is $A^2/2$, in this case $2V^2$. The power of the noise is given as $1V^2$. The total power of the input to the filter is thus $3V^2$. Using the simple formula in the lecture notes and setting $N = 10$ the maximum value of μ for convergence is $\frac{1}{3 \cdot 10 \cdot 3} = 0.033$. Thus

$$0 < \mu < 0.033$$