Solutions to Assignment 1
Sampling and Convolution
(Revised Version)

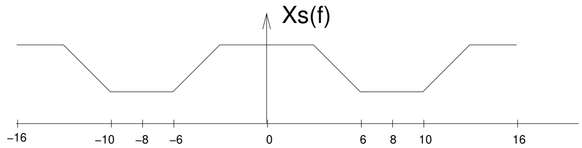
Question 1 - Problem 1.2

The signal has a bandwidth of 10 kHz and the minimum sampling frequency to avoid aliasing will be twice this, or 20 kHz.

If the signal is sampled at 16 kHz the portions of the signal above 8 kHz (the folding frequency) will be aliased. The original and aliased components will overlap in the frequency range of 6 to 10 kHz.

In the overlap region the positive and negative frequency components will add up. The spectrum of a signal is complex. Unfortunately, Figure 1.31 does not indicate whether it is the real, imaginary, magnitude, or phase components that are shown.

If we assume Figure 1.31 shows the magnitude of the spectrum (i.e. $|X(f)|$) and that the phase is zero at all frequencies then the sum of the negative and positive signal portions of the components will be a constant over the overlap range as shown below



Question 2 - Problem 1.9

An 8-bit ADC set up to quantize over the range $+A$ to $-A$ has an rms quantization error voltage of

$$\frac{(2A/2^B)}{\sqrt{12}}$$

We will assume that we want to find the ratio of signal power to quantization power for an input signal which is a sinusoid. Note a signal's power spectrum does *not* define its probability distribution and thus we cannot deduce either its peak value or the aliasing error of the signal in the passband simply from a knowledge of its spectrum.

A sinusoid of frequency f and amplitude A at the input of the anti-aliasing filter will appear at the output with an rms voltage of

$$\frac{A}{\sqrt{2}}\frac{1}{\sqrt{1+(f/f_c)^6}}$$

The filter passband extends from 0 to 3.4 kHz. We need to find the sampling frequency so that the level of a signal aliased to 3.4 kHz will have the same level as the quantization noise.

If we set the two levels equal and solve for f,

$$f = f_c\left[1.5\times 2^{2B}-1\right]^{(1/6)}$$

With $B=8$ and $f_c = 3.4$ we find $f = 23.1$ kHz. So the sampling frequency must be set to 26.5 kHz so that a signal component at 23 kHz, aliased down to 3.4 kHz and have the same level as the quantization noise.

If we wanted the maximum aliasing level over the range 0 to $F_s/2$ to be the same as the quantization noise level then we would set $F_s = 46$ kHz.

If we follow the approach in the text in Example 1.2, we find that $A_{\text{min}} = 20\log(\sqrt{1.5}\times 2^B) = 50$ dB (note the error in the text in the equation for A_{min}). Setting $A_{\text{min}} = 20\log[1+(f/f_c)^6]^{1/2}$ we find $f = 23.1$ kHz. Thus we would set the sampling frequency to $23.1+3.4 = 26.5$ kHz.

Question 3 - Problem 1.10

The quantization step size is

$$V_{fs}/(2^B-1) = 5/(2^{16}-1) = 839\mu\text{V}$$

The rms signal to quantization noise ratio (SQNR) is given by equation 1.11:

$$6.02B + 1.76\text{dB} = 98\text{dB}$$

Question 4 - Problem 1.15

The wording of this question leaves a bit to be desired. Assume the spectrum in the figure is the spectrum that would result if the samples at the output of the DSP system were passed through an ideal reconstruction filter. That is, the digital signal consists of samples of the sum of two sinusoids, one at 1 kHz and one at 3 kHz. Furthermore, assume that $y(n)$ is real so that the spectrum is symmetrical about the origin.

First, since the signal is sampled, the spectrum will repeat at multiples of the sampling frequency, 15 kHz. Thus there will be signal components at $nF_s \pm 1$, and $nF_s \pm 3$ where n is all possible integers including zero. Only the components at 0+1, 0+3, 15-3, 15-1, 15+1, 15+3, 30-3, and 30-1 (1, 3, 12, 14, 16, 18, 27 and 29 kHz) lie in the range of interest (0 to $2F_s$).

Second, due to the zero-order hold processing of the samples by a DAC, the spectrum will be modified by a transfer function equal to the FT of the zero-order hold impulse response:

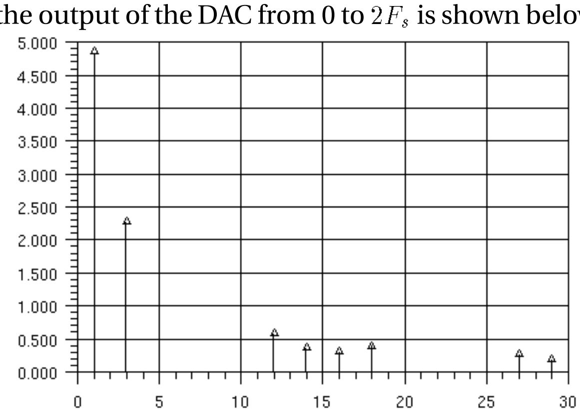
$$|H(f)| = \frac{\sin(2\pi T f)}{2\pi T f}$$

where $T = 1/15$ kHz.

Using the frequencies of interest and scaling the original amplitudes by $|H(f)|$ we obtain:

f	original amplitude	$\lvert H(f)\rvert$	final amplitude
1	5	0.971	4.855
3	3	0.757	2.271
12	3	0.189	0.568
14	5	0.069	0.347
16	5	0.061	0.303
18	3	0.126	0.378
27	3	0.084	0.252
29	5	0.034	0.168

A plot showing the magnitude of the spectrum at the output of the DAC from 0 to $2F_s$ is shown below:



Question 5

The equation defining the convolution sum is:

$$y(n) = h(n) * x(n) = \sum_{k=-\infty}^{\infty} h(k)x(n-k)$$

There are four values of k for which the convolution sum is non-zero: 0, 1, 2, 3.

For $n = 0$				
k	$x(k)$	$n-k$	$h(n-k)$	$x(k) \times h(n-k)$
0	1	0	1	1
1	2	-1	0	0
2	1	-2	0	0
				$\sum = 1$

For $n = 1$				
k	$x(k)$	$n-k$	$h(n-k)$	$x(k) \times h(n-k)$
0	1	1	2	2
1	2	0	1	2
2	1	-1	0	0
				$\sum = 4$

For $n = 2$				
k	$x(k)$	$n-k$	$h(n-k)$	$x(k) \times h(n-k)$
0	1	2	0	0
1	2	1	2	4
2	1	0	1	1
				$\sum = 5$

For $n = 3$				
k	$x(k)$	$n-k$	$h(n-k)$	$x(k) \times h(n-k)$
0	1	3	0	0
1	2	2	0	0
2	1	1	2	2
				$\sum = 2$

The Matlab commands to compute the convolution and plot the results to a file in Postsript format are:

```
>> conv([1 2 1],[1 2])

ans =

     1     4     5     2

>> plot (ans)
```

This plot shows index values from 1 to 4 since Matlab vector indices start at 1. If we wanted to plot the result versus values of n we would have to give the `plot` function both x and y vectors:

```
>> plot(0:3,ans)
>> print -deps sol1.q5.eps
>> quit
```

to get: