Sample Final Exam Questions

These exercises are typical final exam questions. These questions only cover the material from the second part of the course (since the mid-term exam) although the final exam will cover all of the course material.
Solutions will be posted on the Web page on Monday, April 15.

FIR Design - Window Method

A low-pass FIR filter is to be designed using the window method. The sampling rate is 10 kHz, the filter should have low attenuation at frequencies below 3 kHz and the minimum attenuation for frequencies above 4 kHz should be 40 dB.

(a) Sketch the ideal frequency response for this filter.

(b) Give the expression for the impulse response of the ideal filter.

(c) If the impulse response was truncated by windowing, which of the simple windows (rectangular, triangular, Hanning, or Hamming) would produce the filter with the most efficient (fewest operations) implementation that would still meet the specifications?

(d) What is the minimum number of filter coefficients required to meet the specification and what is the duration of this filter's impulse response (in microseconds)? Show your work.

(e) Is this a linear-phase filter?

The bilinear z-transform

An analog filter has the following transfer function (where a is a constant):

$$H(s) = \frac{1}{as+1}$$

(a) What is the -3 dB frequency of the above filter (in radians per second)?

(b) We would like to design an equivalent digital filter using using the bilinear z-transform. If the sampling rate of this filter will be 10 Hz and the digital filter must have a -3 dB frequency of 5 Hz, what should the -3 dB frequency of the analog filter be? What should the value of a be?

(c) Using the value computed above for a, use the bilinear z-transform to obtain the transfer function and signal flow graph for the digital filter with a -3 dB frequency of 5 Hz.

Finite Precision Filter Implementation

(a) Draw the direct form signal flow graph that implements the following IIR filter:

$$H(z) = \frac{z^2+2z+1}{z^2+2z-3}$$

(b) Design and draw the signal flow graph for an equivalent filter composed of a two first-order sections in parallel. Show your work.

Decimation and Interpolation

(a) A signal composed of two sinuoids at frequencies of 0.5 and 2 kHz is sampled at 8 kHz. The sampling rate is then reduced to 2 kHz by taking every fourth sample. Draw the amplitude spectrum of the resulting decimated signal from -2 to 2 kHz. Indicate on the sketch the frequencies and relative levels of all spectral components.

(b) A noise-like signal with a spectrum that is flat from 0 to 30 kHz and zero elsewhere is sampled at 100 kHz. Sketch the spectrum that would result if two zeroes are now inserted between each sample, effectively increasing the sampling rate by a factor of 3. Include components at frequencies from -350 to 350 kHz in the sketch (not necessarily to scale). Indicate on the sketch the frequencies and relative levels of all spectral components.

Spectral Estimation

The Welch method is to be used to compute the spectrum of a random signal sampled at 10 kHz. Assume that resolution is defined as the frequency of the first zero of the Fourier Transform of the window function. How many samples will be required to obtain a resolution of 100 Hz if a rectangular (i.e. no) window is used? If a 50% overlap is used, approximately how many samples are required so that 7 periodograms can be averaged?

Adaptive Filtering

It is desired to remove an interfering sinusoid from a (uncorrelated) noise-like signal.

(a) Draw a diagram of an adaptive filter that could accomplish this task. The diagram should have one input (for the noisy sinusoid) and two outputs (extracted sinusoid and extracted noise). Label each block in the diagram with its function.

(b) If the sinusoid's amplitude is 2 volts and the noise variance is 1 V^2, use the simple bound given in the lecture notes to find the possible range of values of μ for a 10-tap adaptive filter using the LMS algorithm.