Lecture 9 - Error Detection and Correction

$\boxed{1\ 0}$

$$\frac{6}{4} = 2 \text{ rem } 2$$

Exercise 1: Compute the modulo-4 checksum, *C*, of a frame with byte values 3, 1, and 2. What values would be transmitted in the packet? What would be the value of the checksum at the receiver if there were no errors? Determine the checksum if the received frame was: 3, 1, 1, *C*? 3, 1, 2, 0, *C*? 1, 2, 3, *C*?

$3+1+2 = 6$ $\quad c = -2$

$3, 1, 2, -2 \Leftarrow$ transmit this.

$3+1+2-2 = 4$ $\quad$ 4 modulo 4 = 0 ✓ no errors.

$3+1+1+-2 = 3$ $\quad$ 3 modulo 4 = 3 X error

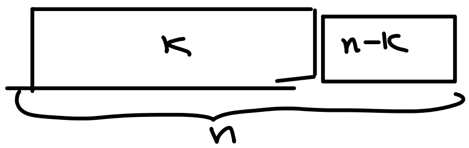
$3+1+2+0+-2 = 4$ $\quad$ 4 modulo 4 = 0 ✓ no errors

$3+2+1+-2 = 4$ $\quad$ 4 modulo 4 = 0 ✓ no errors

Exercise 2: What is a modulo-2 sum? What is the modulo-2 sum of 1, 0 and 1? What is the modulo-2 sum if the number of 1's is an even number?

- Modulo-2 = remainder after divide by 2
  - even (0) or odd (1)
  - L.S. bit (1 - odd, 0 - even)
- $1+0+1 = 2$ $\quad$ 2 mod 2 = 0
- if even number of 1's → modulo 2 sum is zero

Exercise 3: How many possible code words are there for an (n, k) code? How many possible parity bit patterns are possible for each code word?

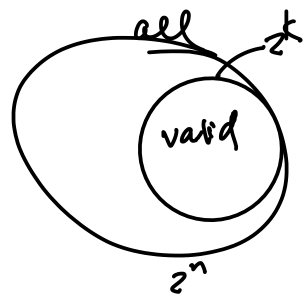


codewort is a block of n bits

only 2^k possible data values $\therefore$ 2^k valid code words

2^k valid code works

typically only one valid setg n-k parity bits per codeword.



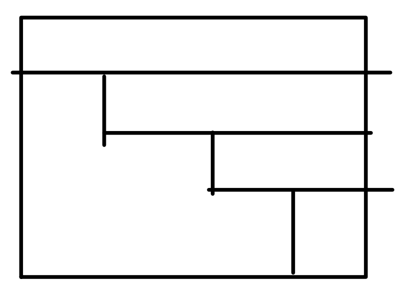
Exercise 4: What is the Hamming distance between the codewords 11100 and 11011? What is the minimum distance of a code with the four codewords 0111, 1011, 1101, 1110?

$$\begin{array}{r} 11100 \\ 11011 \\ \hline 1+1+1 = 3 \end{array}$$

2	2	2
	2	2
		2

minimum = 2

$D_{min} = 2$



Exercise 5: A block code has two valid codewords, 101 and 010. The receiver receives the codeword 110. What is the Hamming distance between the received codeword and each of the valid codewords? What codeword should the receiver decide was sent? What bit was most likely in error? Is it possible that the other codeword was sent?

valid	received	Hamming
101	110	2
010	110	1

← smallest Hamming dist receiver chooses this codeword.

$$\begin{array}{r} 010 \\ \oplus\ 110 \\ \hline 100 \end{array}$$

bit most likely in error ∴ bit that is corrected.

Exercise 6: What is the minimum distance for the code in the previous exercise? How many errors can be detected if you use this code? How many can be corrected? What are n, k, and the code rate (k/n)?

$$\begin{array}{r} 010 \\ \oplus\ 101 \\ \hline 1+1+1 = 3 \end{array}$$

$D_{min} = 3 = d$

$d - 1 = 2$ ∴ up to 2 errors can be detected

$\left\lfloor \frac{d-1}{2} \right\rfloor = \left\lfloor \frac{3-1}{2} \right\rfloor = 1$ ∴ up to 1 errors can be corrected.

Exercise 7: Assume 1000-bit frames are being transmitted at 1 Mb/s. What is the throughput if there are no errors? Now assume errors introduced by the channel cause a frame error rate of 90%. What is the throughput?

Now assume we use a rate-1/2 FEC code. How many bits must be transmitted in each frame? What is the throughput if there are no errors? Now assume that with FEC coding the receiver corrects most of the errors and the frame error rate drops to 1%. What is the new throughput?

Now assume the ~~FEC can only correct 10% of the frames~~, what is throughput? FER is 10%

When is it worthwhile to use FEC? What other advantage might the use of FEC provide?

In all cases above ignore the effect of retransmissions.

FER = frame
BER = bit
error rate

no errors: Throughput = 1 Mb/s.

90% FER ; 10% frames received error-free (usable)

fr

∴ Throughput = 100 kb/s

rate $\frac{1}{2}$ & assume frame size increases to 2000 bits

$n = 2000$, $k = 1000$ $\quad \frac{k}{n} = \frac{1}{2}$ (rate $\frac{1}{2}$ code)

if no errors transmit @ 1 Mb/s 500 frames/second

∴ throughput = 500 kb/s $\left(\begin{array}{c} \frac{1}{2} \cdot 1 \text{ Mb/s} \\ 500 \text{ frames/s} \cdot 1000 \text{ b/frame} \end{array} \right)$

if errors but FEC corrects enough errors so that FER = 1%

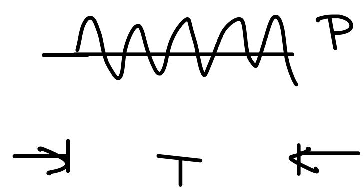
throughput = 99% · 500 kb/s = 495 kb/s

if FER is 90%

throughput = 10% · 500 kb/s = 50 kb/s.

- FEC worthwhile if increases throughput.
- FEC results in shorter delays than retransmission.
- FEC useful if no ability to retransmit (e.g. broadcast ch.)

Exercise 8: What are the units of Energy? Power? Bit Period? How can we compute the energy transmitted during one bit period from the transmit power and bit duration?



Energy : Joules

Power : Watt = Joules/second.

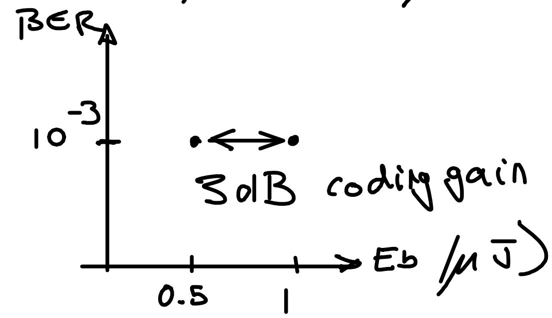
Bit Period: seconds

Energy per bit = Power · bit period

(Joules/s · seconds → Joules)

Exercise 9: A system needs to operate at an error rate of 10^{-3}. Without FEC it is necessary to transmit at 1W at a rate of 1 Mb/s. When a rate-1/2 code is used together with a data rate of 2 Mb/s the power required to yachieve the target BER decreases to 500mW. What is the channel bit rate in each case? What is the information rate in each case? What is E_b in each case? What is the coding gain?

	w/o FEC	w/ FEC
ch. bit rate	1 Mb/s	2 Mb/s
inf. rate	1 Mb/s	1 Mb/s
Eb $(P \cdot T_b)$	$1W \cdot 1\mu s = 1 \mu J$	$0.5W \cdot 1\mu s = 0.5 \mu J$



Exercise 10: Assuming one bit at a time is input into the encoder in the diagram above, what are k, n, K and the code rate?

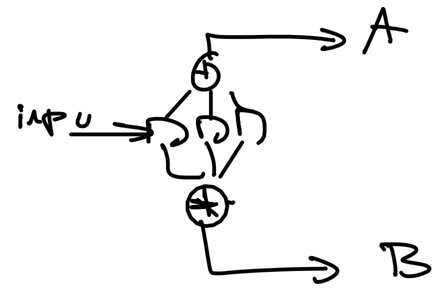
$k = 1$

$n = 2$

$K = 7$

$\text{rate} = \frac{k}{n} = \frac{1}{2}$

Exercise 11: Consider the encoder above. If the only the bits corresponding to the outputs A, A and B, and B are transmitted corresponding to every three input bits, what is the code rate of this punctured code?



bit 0	bit 1	bit 2
A	A	
	B	B

A_0, A_1, B_1, B_2

$$\text{rate} = \frac{3 \text{ input bits}}{4 \text{ output bits}} = \frac{3}{4}$$

Exercise 12: Give the numbering of the bits coming out of a 4x4 interleaver. If bits 8, 9 and 10 of the interleaved sequence have errors, where would the errors appear in the de-interleaved sequence? If the receiver could correct up to one error per 4-bit word, would it be able to correct all the errors without interleaving? With interleaving?

1	2	~~3~~	4
5	6	~~7~~	8
9	10	11	12
13	~~14~~	15	16

1, 5, 9, 13, 2, 6, 10, ~~14~~ OK fixed, ~~3~~, ~~7~~, 11, 15, 4, 8, 12, 16

W/o interleaving one word has errors

W/ interleaving all errors are corrected

Exercise 13: If errors on the channel happened in bursts and you were using a RS code using 8-bit words, would you want to interleave bits or bytes?