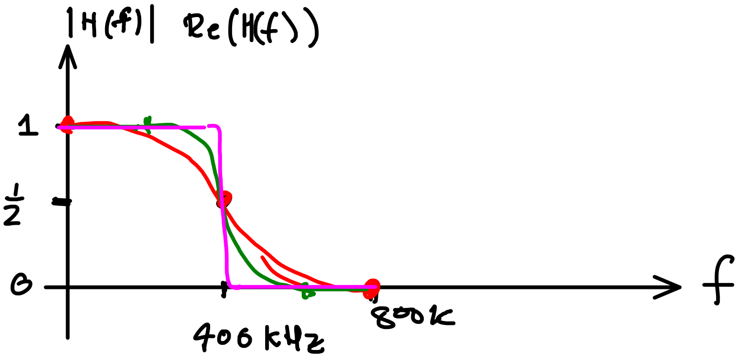
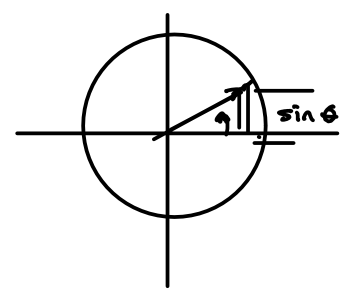
Lecture 5 - Data Transmission over Band-Limited Channels

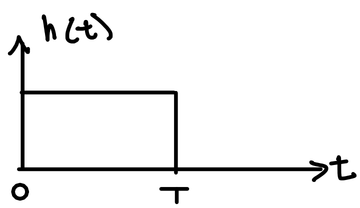
Exercise 1: Draw the (real portion of) a raised-cosine transfer function that would allow transmission of impulses at a rate of 800 kHz with no interference between the impulses.

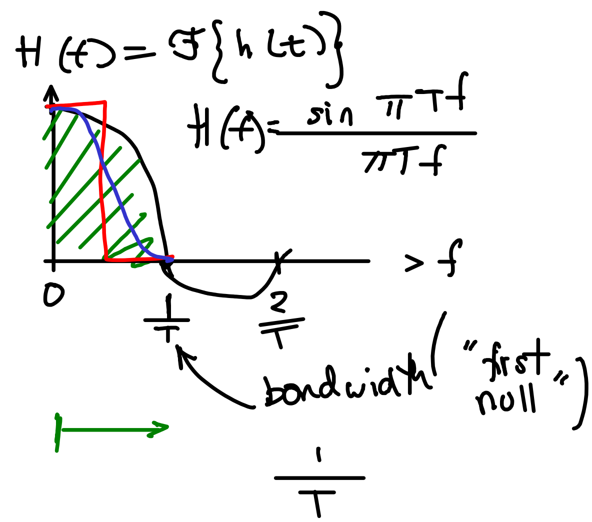
$f_{symbol} = 800$ kHz



Exercise 2: What is the impulse response of a filter than converts input impulses to pulses of duration T? What is the shape of the frequency response of this filter? *Hint: the Fourier transform of a pulse of duration T is* $\frac{\sin(\pi T f)}{\pi T f}$. What is the "bandwidth" of this filter (when is it first zero)? How does this compare to the "bandwidth" of the raised-cosine filter above?







the "first null" bandwidth of the pulse-shaping filter is the same as that of the raised-cosine filter but it rolls off more gradually at lower frequencies.

Exercise 3: What is the possible range of values of α?

0% 100%

$$0 \leq \alpha \leq 1$$

minimum
"brick wall"
from 0 to $\frac{1}{2T}$

maximum value
extends from 0 to $\frac{1}{T}$

Exercise 4: Could equalization be done at the receiver only? At the transmitter only? Why or why not?

yes
yes } in theory

because only overall response has to meet Nyquist,

practically it may be easier to do it at one side or the other e.g. receiver.

if done at TX need to feedback information about channel.

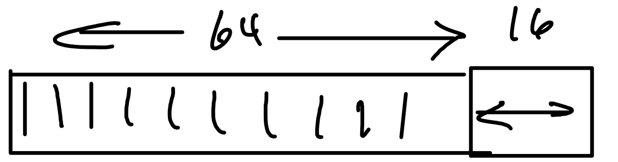
Exercise 5: The 802.11g WLAN standard uses OFDM with a sampling rate of 20 MHz, with $N = 64$ and guard interval of $0.8\mu s$. What is the total duration of each OFDM block, including the guard interval? How many guard samples are used?

$N = 64$

$f_s = 20MHz$

$T_{sample} = \frac{1}{f_{sample}}$

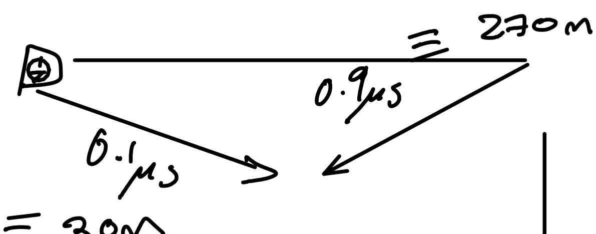
$T_{symbol} = 64 \cdot T_{sample} = 64 \cdot \frac{1}{20 \times 10^6} = 3.2\mu s$



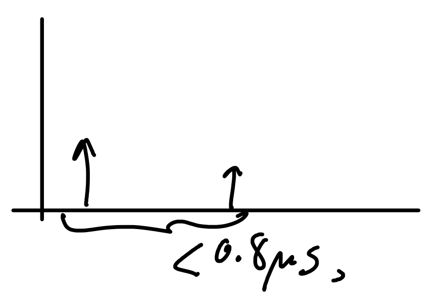
$T_{symbol} = 3.2\mu s$ $0.8\mu s$.

$4.0\mu s$

$N_{guard} = \frac{T_{guard}}{T_{sample}} = \frac{0.8}{50ns} = 16$ samples



$3 \times 10^8 m/s = 300m/\mu s$.



Exercise 6: What is the channel capacity of a 3 kHz channel with an SNR of 20dB?

$B =$

$\frac{S}{N}$

$\log_2(100) = \frac{\log(100)}{\log(2)}$

$C = B \log_2\left(1 + \frac{S}{N}\right) = 3000 \cdot \log_2\left(1 + 10^{\left(\frac{20}{10}\right)}\right)$

$\simeq 3000 \cdot \log_2(100) \simeq 20 kb/s$

Exercise 7: What are some differences between the signalling rate limit imposed by the Nyquist no-ISI criteria and the Shannon Capacity Theorem?

	Nyquist	Shannon
limits:	symbol rate w/o ISI	information rate at arbitrarily low error rate
units:	symbols/s	bits/s.
function of	transfer function	Bandwidth & $\frac{S}{N}$ ratio