

## Assignment 3 Solutions

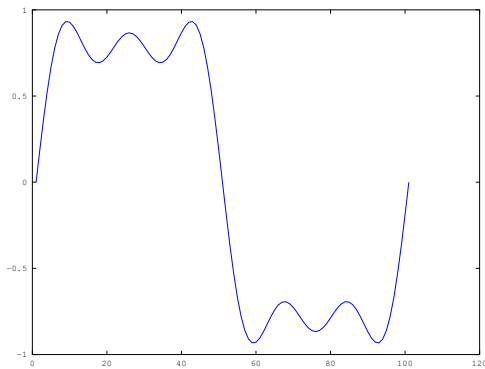
### Question 1

The Matlab<sup>1</sup> statements and the resulting waveforms at the output of the channel are as follows:

- (a) The statement to produce the input waveform is:

```
plot(sin(2*pi*t)+ ...  
1/3*sin(2*pi*3*t)+1/5*sin(2*pi*5*t))
```

and the undistorted output looks somewhat like a pulse:

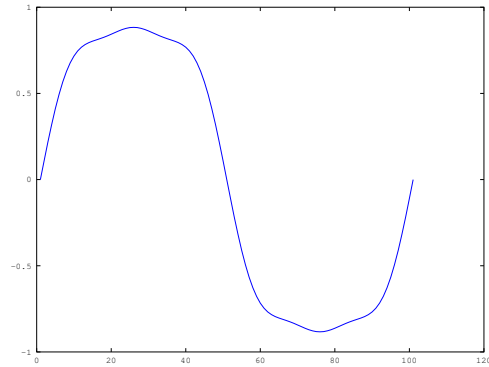


- (b) 6 and 12 dB attenuations correspond to voltage factors of 2 and 4 respectively. So the resulting waveform can be computed and plotted with the following statement:

```
plot(sin(2*pi*t)+...  
1/3/2*sin(2*pi*3*t)+...  
1/5/4*sin(2*pi*5*t))
```

and the output looks more like the fundamental sine wave because of the attenuation of the higher-frequency components:

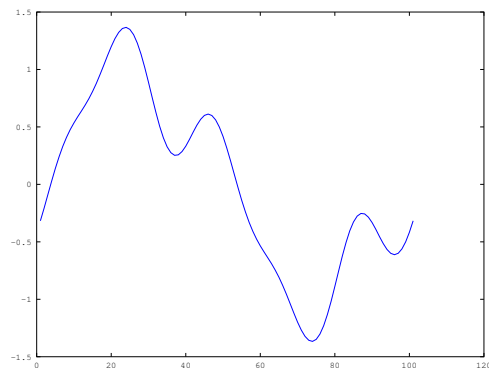
<sup>1</sup>octave was used for these examples.



- (c) A delay can be produced by subtracting<sup>2</sup> the appropriate amount from each time argument:

```
plot(sin(2*pi*t)+...  
1/3*sin(2*pi*3*(t-0.1))+...  
1/5*sin(2*pi*5*(t-0.2)))
```

and the different delays for the different frequency components (i.e. a channel with non-linear phase) results in a distorted waveform:



### Question 2

- (a) It's difficult to measure the exact bandwidths from the graphs, but in the transition region the filter rolls off approximately linearly between 1 kHz and 1.5 kHz from 0 to -70 dB. This is a

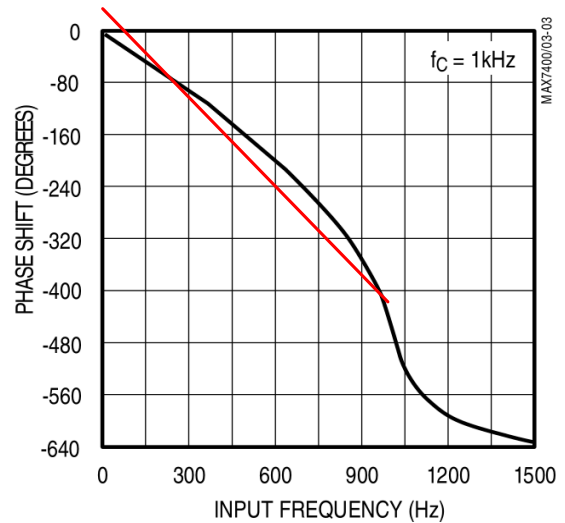
<sup>2</sup>In Matlab, subtracting a scalar from a vector subtracts that scalar from each element of the vector.

slope of  $-140$  dB/kHz. From this we can compute the  $-3$  dB and  $-20$  dB frequencies (and thus bandwidths since the response extends to DC).

The response will be  $-3$  dB at an offset of  $1 + \frac{3}{140} \times 1 = 1.02$  kHz.

- (b) Similarly, the response is  $-20$  dB at an offset of  $1 + \frac{20}{140} \times 1 = 1.14$  kHz. This is the  $-20$  dB bandwidth
- (c) As computed above, the roll-off in the transition between passband and stopband would be  $140$  dB between  $1$  and  $2$  kHz, which is one octave.
- (d) The minimum stopband attenuation can be read from the first graph as about  $-82$  dB.
- (e) The passband ripple can be read from the second graph. The gain in the passband varies from about  $0.04$  dB to  $0.22$  dB or a total of about  $0.18$  dB.
- (f) The phase shift at  $500$  Hz can be read from the third graph as  $-160^\circ$ . The delay can be computed from the equation  $\theta = 360^\circ \times \tau f$  using  $f = 500$  and  $\theta = 160$ . The delay is  $\tau = \frac{\theta}{360 \times f} = 890 \mu\text{s}$ .
- (g) The (approximate) maximum deviation from linear phase over the frequency range  $0$ - $1$  kHz can be obtained by fitting a straight line that has the minimum maximum (“minimax”) deviation from the data and then computing or measuring this deviation.

A simple approximation can be made by estimating this line by drawing a straight line that appears to have the same maximum phase deviation above and below the line. The diagram below shows that the filter’s phase response deviates about  $\pm 40^\circ$  from linear phase over the range  $0$  to  $1$  kHz.



### Question 3

- (a) The noise power at the output of an amplifier with gain  $G$  is  $P = -174 + 10 \log(B) + 10 \log(F) + 10 \log(G)$ . We are given  $G = 20$  dB,  $B = 60$  dB-Hz and  $P = -91$  dBm so we can solve for  $F = P + 174 - 10 \log(G) - 10 \log(B) = -91 + 174 - 20 - 60 = 3$  db.
- (b) The apparent noise power at the amplifier input is  $20$  dB (the gain) less than the power at the output or  $-91 - 20 = -111$  dBm or  $10^{-11.1} = 7.9 \times 10^{-12}$  mW  $= 7.9 \times 10^{-15}$  W. This power is  $N = kTBF$ . Setting  $F = 1$ ,  $B = 10^6$  Hz, and  $k = 1.4 \times 10^{-23}$  J/K,

$$T = \frac{N}{kBF} = \frac{7.9 \times 10^{-15}}{1.4 \times 10^{-23} \times 10^6 \times 1} = 564 \text{ K}$$

**Important Note:** While this is the temperature of a resistor that would generate the observed noise power, it is *not* the correct definition for the noise temperature of an amplifier. Instead, the noise temperature of an amplifier is defined as the temperature of resistor that would create the *additional* noise added by the amplifier over and above the noise created by a resistor at the input at a temperature of  $290$  K. A perfect amplifier, one that adds no noise of its own, thus has a noise temperature of  $0$  K. However this perfect amplifier would still output noise if there was any noise present at its input.

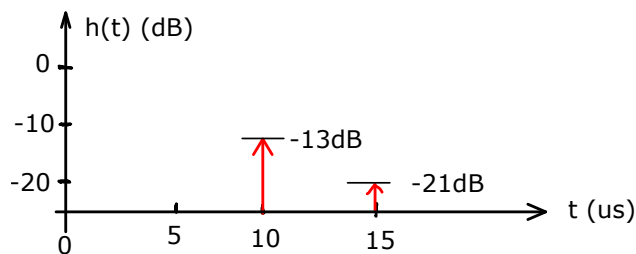
Thus the noise temperature is numerically equal to the temperature computed above minus 290 K, or  $564 - 290 = 274$  K. Because of the error in the question, either answer will be marked correct.

#### Question 4

The impulse travels down the 2 km line in  $10 \mu\text{s}$  but is attenuated by 3 dB by the first splitter and by 10 dB by the line.

The reflection from the tap travels a total of 3 km so it arrives after  $15 \mu\text{s}$  and is attenuated by 3 dB by the first tap, 3 dB by the second and 15 dB by the line.

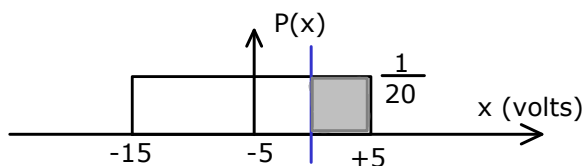
The overall impulse response is thus a direct component at a delay of  $10 \mu\text{s}$  with a level of  $-13$  dB and a multipath component with a delay of  $15 \mu\text{s}$  and a level of  $-21$  dB:



The diagram showed the reflection coefficient  $\Gamma$  with a value of  $-1$  so a reflection with the opposite polarity would also be marked correct.

#### Question 5

(a) RS-232 uses the negative voltage ( $-5\text{V}$  in this case) to transmit a '1'. When this voltage is added to noise, every received value will have  $-5\text{V}$  added to it. Thus the probability density will shift down (to the left) by  $5\text{V}$ :



A reasonably-designed receiver will decide a '0' was sent (that is, make an error) if the received

signal level is positive. Thus the receiver will make an error whenever the signal plus noise is greater than  $0\text{V}$ .

The probability of a random variable being within a range of values is the area under the probability density curve between those values (the shaded area above). In this case the probability of the signal being greater than  $0\text{V}$  is the area of the curve to the right of  $0\text{V}$ .

Since this is a rectangle the area is the width times the height:  $5 \times \frac{1}{20} = 0.25$ .

(b) If the noise has a Gaussian distribution with a mean of  $0\text{V}$  and a standard deviation of  $3\text{V}$ , the probability that the noise is less than a threshold value of  $5\text{V}$  can be found by first computing the value of  $t = \frac{5-0}{3} = 1.66$ . Using a calculator, tables or the approximation formula,  $P(1.66) = 0.95$ . The probability that the noise is greater than  $5\text{V}$  is thus  $1 - 0.95 = 5\%$ .

The probability of error if a '1' bit is transmitted can be computed as in part (a) above. The only thing that changes is the shape of the curve. The mean ( $\mu$ ) of the signal plus noise now becomes  $-5\text{V}$  instead of  $0$ . The standard deviation ( $\sigma$ ) is unchanged at  $3\text{V}$ .

To compute the area under the curve to right of  $0\text{V}$  we have to first compute the area to the left of  $0\text{V}$  using the Gaussian CDF and then subtract that value from 1 to find the area to the right of  $0\text{V}$ .

For a threshold of  $0$ , a mean of  $-5$  and a standard deviation of  $3$  the value of  $t = \frac{0-(-5)}{3} = 1.66$ . Using a calculator, tables or the approximation formula,  $P(1.66) = 0.95$ . The probability that the signal plus noise is greater than  $0\text{V}$ , that is, the probability of an error or the Bit Error Rate (BER), is thus  $1 - 0.95 = 5\%$ .