ELEX 4340 : Data Communication
2014 Fall Session

Solutions to Assignment 2

With corrections as of October 15.

Question 1

(a) Using

$$\varepsilon_r = \frac{1}{VF^2}$$

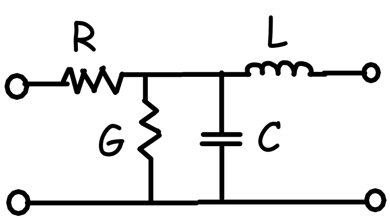
and $VF = 0.70$ for the Cat-5 cable and 0.84 for the co-ax, $\varepsilon_r = 2.0$ and 1.4 respectively

The value of ε_r is a function of the dielectric and the data sheet says the co-ax dielectric is “gas-injected foam HDPE insulation.” The gas (probably nitrogen) has ε_r close to 1 resulting in a lower~~higher~~ overall dielectric constant than for a dielectric composed solely of polyethylene.

(b) Table 1 in the datasheet for the UTP cable shows the attenuation at 100 MHz is 22 dB per 100 m. For the co-ax the datasheet has specifications per foot. The ratio of meters to feet is $\frac{1000/25.4}{12} = 3.28$. The co-ax attenuation at 100 MHz is 1.3 dB per 100 ft or $1.3 \times 3.28 = 4.3$ dB per 100 m.

(c) Since the frequency response extends to DC, the 3 dB bandwidth is the frequency at which the loss of a 100 m length of the cable is 3 dB or $3/3.28 = 0.91$ dB per 100 ft. For the UTP, we can interpolate between values of 1 MHz at 2 dB and 4 MHz at 4.1 dB. A straight line fit is $f = 1 + \frac{4-1}{4.1-2}(A-2)$ where A is the attenuation in dB and f is the frequency in MHz. For an attenuation of 3 dB $f = 1 + 1.4 \times (3-2) = 2.4$ MHz. Similarly, for the co-ax $f = 50 + \frac{100-50}{1.3-0.9}(A-0.9)$ and for $A = 0.91$ the 3 dB bandwidth is $f = 51$ MHz.

(d) The equivalent lumped-element circuit is:



For the UTP cable the capacitance is specified as $C = 15$ pF/ft or $15 \times 3.28 = 49$ pF/m. The inductance was specified in the question as $L = 525$ nH/m. The DC resistance (per conductor) is given as 9.38 Ohms/100m so the resistance in the lumped-element model is $R = 2 \times 9.38/100 = 0.19$ Ohms/m.

For the co-ax the datasheet gives the capacitance as $C = 16.1$ pF/ft or $16.1 \times 3.28 = 53$ pF/m, the inductance as $L = 97$ nH/ft or $97 \times 3.28 = 318$ nH/m, and the DC resistance is the sum of the center conductor and shield resistances or $2.6 + 3 = 5.6$ Ohms/1000 ft or 0.018 Ohms/m.

(e) For $\varepsilon_r = 1.4$ as calculated above, an inner conductor diameter of $d = 0.064$ and shield diameter of $D = 0.280$ (inches) the simplified equations given in the lecture notes predicts a characteristic impedance of

$$Z_0 \approx \frac{60}{\sqrt{1.42}} \ln\left(\frac{0.280}{0.064}\right) = 74\text{Ohm}$$

This is slightly (about 1%) less than the cable specification (75 ohms).

Question 2

For Belden GCAC fibre-optic cable using single-mode 9/125 G.655 fiber or the multi-mode 62.5/125 multi-mode fiber:

(a) the maximum dimensions of the core/cladding are 9/126 and 65/126 μm respectively.

(b) The maximum loss per km at the lowest-loss wavelength (1550nm and 1300nm respectively) are 0.30 and 1.1 dB/km respectively. The RG-11 co-ax has a loss of $1.3 \times 3.28 = 4.3$ dB/100 m or 42.6 dB/km at 100 MHz. Thus the loss of either fiber optic cable is much lower. For example, the single-mode cable could be run $42/0.3 \approx 140$

times further for the same attenuation. This comparison ignores other issues that are often more important such as the available bandwidth and the costs of the cable, installation and interfaces.

Question 3

A signal with a voltage of 300 mVrms is measured at the receiving end of a transmission line that has a nominal impedance of 600 Ω. Thus the received power is $P = \frac{V^2}{R} = \frac{0.3^2}{600} = 150 \times 10^{-6}$ W $= 150 \times 10^{-3}$ mW or $10 \log(150 \times 10^{-3}) = -8.2$ dBm. If the loss of the transmission line is known to be 5 dB/100m at the signal frequency, and the power at the transmitting side was +20 dBm the length of the line would be $\frac{20-(-8.2)}{5} \times 100 = 560$ m.

Question 4

If 0-gauge (zero gauge) AWG wire has a diameter of about 8.3 mm and the wire diameter is reduced by a factor of about 0.891 for each increase of 1 in wire gauge, the diameter of 1-gauge wire is $8.3 \times 0.891 =$ 7.4~~6.~~6 mm. The diameter of 2-gauge wire would be $8.3 \times 0.891 \times 0.891 = 8.3 \times 0.891^2 =$ 6.6~~5.~~4 mm. The diameter of n-gauge wire would be 8.3×0.891^n mm.

Question 5

(a) The surface area of a sphere of radius d is:

$$A = 4\pi d^2 \frac{1}{\cancel{4\pi d^2}}$$

(b) If we transmit P_T Watts from an antenna that radiates equally well in all directions (an "omnidirectional" antenna), then power density (in W/m^2) at distance d from the antenna is

$$\frac{P_T}{A} = \frac{P_T}{4\pi d^2}$$

(c) If a transmitting antenna is made directional so that it concentrates all its power into a fraction of the sphere that is $1/G_T$ of the total area, then the power density in the "illuminated" area will be higher by a factor G_T or:

$$\frac{P_T G_T}{4\pi d^2}$$

(d) If a receiving antenna collects all of the power "shining" on an area A_e on the surface of this sphere (the "effective area"), the power collected (that is, received) is the power density times the area or:

$$P_R = \frac{A_e P_T G_T}{4\pi d^2}$$

(e) Setting this power equal to the received power predicted by the Friis equation:

$$P_R = P_T G_T G_R \left(\frac{\lambda}{4\pi d}\right)^2$$

we have:

$$\frac{A_e P_T G_T}{4\pi d^2} = P_T G_T G_R \left(\frac{\lambda}{4\pi d}\right)^2$$

or, canceling common terms:

$$A_e = G_R \frac{\lambda^2}{4\pi}$$

(f) At 2.4 GHz the wavelength $\lambda = c/f = \frac{3\times10^8}{2.4\times10^9} =$ 0.125 m. The effective area of an omnidirectional ($G_R = 1$) is thus:

$$A_e = 1\frac{0.125^2}{4\pi} = 1.2 \times 10^{-3}\text{m}^2 = 12.4\ \text{cm}^2$$

(g) At 12 GHz the wavelength $\lambda = c/f = \frac{3\times10^8}{12\times10^9} =$ 2.5 cm. The effective area of an 30 dB ($G = 1000$) antenna is thus:

$$A_e = 1000\frac{0.025^2}{4\pi} = 0.05\ \text{m}^2 = 500\text{cm}^2$$

which would be a circle of about 25 cm diameter.

The term "omnidirectional" is sometimes used to mean that the antenna radiates equally in all azimuths (all horizontal directions) rather than uniformly in both azimuth and elevation. "Isotropic" is the formal term for the latter.