ELEX 4340 Lecture 9 Notes

Exercise 1: Compute the modulo-4 (3-bit) checksum of a frame with values 4, 1, and 3. Would an error be detected if the received frame was 4, 1, 3, 0? How about if the received frame was 1, 4, 3?

$$\begin{array}{r} 4 \\ 1 \\ +\ 3 \\ \hline =\ 8 \end{array}$$

8 mod 4 = 0

$$\begin{array}{r} 4 \\ 1 \\ 3 \\ +\ 0 \\ \hline =\ 8 \end{array}$$

8 mod 4 = 0

NO

$$\begin{array}{r} 1 \\ 4 \\ +\ 3 \\ \hline =\ 8 \end{array}$$

8 modulo 4 = 0

NO

Exercise 2: What is a modulo-2 sum? What is the modulo-2 sum of 1, 0 and 1? What is the modulo-2 sum if the number of 1's is an even number?

- modulo -2 sum is remainder after dividing by 2.
- 1 + 0 + 1 = 2

  2 modulo 2 = 0   mod(2, 2)

  in C: 2 % 2 = 0.   or 2 & 1 = 0
- if even number < ls. bit is 0

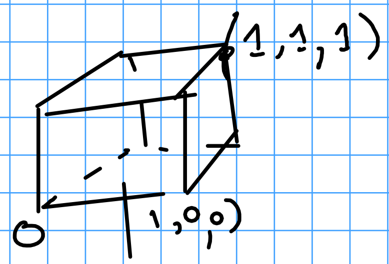
  modulo 2 value is 0

Exercise 3: How many different code words (different blocks) does an (n, k) code have? How many different patterns of $n - k$ parity bits are there?

2^K different code words for an (n, k) code.

2^{n-k} different parity bit patterns of length $n-k$

Exercise 4: What is the Hamming distance between the code-words 11100 and 11011?



$$\begin{array}{rccccc} & 1 & 1 & 1 & 0 & 0 \\ \overset{\wedge}{(\text{xor})} & 1 & 1 & 0 & 1 & 1 \\ \hline \end{array}$$

$$\sum 0+0+1+1+1 \rightarrow 3$$

Hamming distance is 3. (3 bits differ)

Exercise 5: A block code has two valid codewords, 101 and 010. The receiver receives the codeword 110. What is the Hamming distance between the received codeword and each of the valid codewords? What codeword should the received decide was transmitted?

$$\rightarrow 1\,0\,1$$

$$\rightarrow \underbrace{\underbrace{0}_{k}\ \underbrace{1\,0}_{n-k}}_{n}$$

$n = 3$ (block size)

$k = 1$ (data bit)

$n-k = 2$ parity bits

receive: 110

distance from 101 :

$$\begin{array}{r} 110 \\ \oplus\ 101 \\ \hline \Sigma\ 0,1,1 \end{array} \Rightarrow \text{distance} = 2$$

from 010 :

$$\begin{array}{r} 110 \\ \oplus\ 010 \\ \hline \Sigma\ 1,0,0 \end{array} \Rightarrow \text{distance} = 1$$

choose 010 as best guess for transmitted code word

∴ the error was

$$\begin{array}{r} 110 \\ \oplus\ 010 \\ \hline 100 \end{array}$$

error ↖ location

Exercise 6: Assume errors on a channel cause a frame error rate of 50%. When a rate-1/2 FEC code is used the frame error rate drops to 2%. Compute the throughputs with and without coding relative to the uncoded and error-free channel. What other advantages might the use of FEC provide?

throughputs:

error free : rate B

no FEC : $50\% \times B = 0.5B$

rate $\frac{1}{2}$ FEC : $0.5 \cdot B \cdot 0.98 = 0.49B$

Warning: 1/ does not include effect of retransmissions (not realistic)

2/ does not consider delay..

Other advantage : REDUCED DELAY.

Watts × seconds = Joules.

e.g. transmitter uses 1W to transmit $\underbrace{10\ \text{Mb/s}}$

how many Joles/bit? bit duration = 100ns

E_b = 1 W · 100ns = 0.1 μJ

Energy/bit

Exercise 7: A system without coding needs to transmit at 1W to transmit 1 Mb/s at an error rate of 10^{-3}. When a rate-1/2 code is used the power to transmit the necessary 2Mb/s of data and parity bits decreases to 500mW. What is the channel bit rate in each case? What is the information rate in each case? What is E_b? What is the coding gain?

	without coding	with rate ½ coding
Power	1 W	½ W
Error Rate	10^{-3}	10^{-3}
Ch. Data Rate	1 Mb/s	2 Mb/s
Information Rate	1 Mb/s	1 Mb/s
E_b (per infor. bit)	1 μJ/bit	½ μJ/bit

coding gain = 3 dB.

Exercise 8: Assuming one bit at a time is input into the encoder in the diagram above, what are k, n, K and the code rate?

$k = 1$ input bit at a time

$n = 2$ output " " "

$K = 7$ bits affect the output

R = rate = $\frac{k}{n} = \frac{1}{2}$

Exercise 9: Consider the encoder above. If the only the bits corresponding to the outputs A, A and B, and B are transmitted corresponding to every three input bits, what is the code rate of this punctured code?

$1 \rightarrow A_1, B_1$

$2 \rightarrow A_2, B_2$

$3 \rightarrow A_3, B_3$

3 input bits → 6 output bits

if only transmit A_1, A_2, B_2, B_3 ← "puncturing"

$$\text{code rate} = \frac{3 \text{ bit of data}}{4 \text{ bits sent}} = \frac{3}{4}$$