Lecture 7 - Answers to Lecture Exercises

Exercise 1: Approximately what frequencies are used by each of the following: Telephones? AM broadcasting? Ethernet LAN? Cable TV? Which are baseband channels?

Telephone: Baseband ≈ 300 Hz to 4 kHz

AM Broadcasting: Passband $f_c \approx$ 600 kHz → 1.5 MHz

$B \approx$ 10 kHz

Ethernet LAN: Baseband up to ≈ 100 MHz
≈ 350?

Cable TV:

> 50 MHz to 100's of MHz → to subscriber

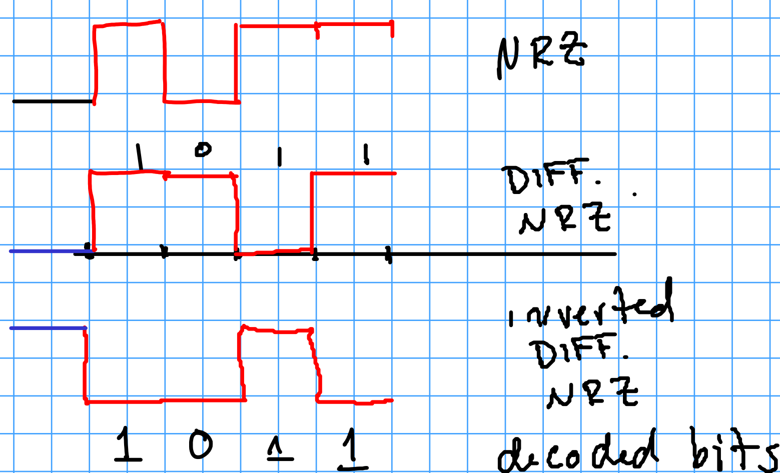
← to head end < 50 MHz

bandwidth ≈ 5 MHz (TV signal)

Exercise 2: Assume a 1 is transmitted as 5V and 0 as 0V. Draw the waveform for the bit sequence 1011. Draw the waveform if the bits are transmitted differentially with a 1 encoded as a change in level. Assume the initial value of the waveform is 0. Invert the waveform and decode it.

1011

→ 110



Exercise 3: You receive the sequence of bits 10001101 and are told that bit stuffing was used to limit runs of 0 to three or fewer. What sequence is the original data sequence?

transmit data → 1 0 0 0 1 0 1

stuffed bit ↓

1 0 0 0 ~~1~~ 1 0 1

count 0 1 2 3 0 0 1 0 ← on the channel

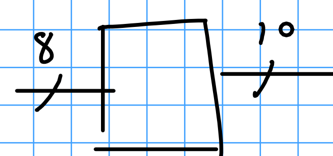
received data → 1 0 0 0 1 0 1

Exercise 4: How many combinations are there of 3 bits? Of 4 bits? How many bits might be input and output by an 8B10B code? What might a 4B3T code mean?

$2^3 = 8$

$2^4 = 16$

8B10B: 8 binary input (bits), 10 binary output (2 levels)

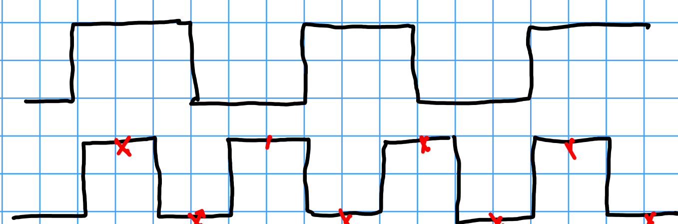


2 : BINARY

3 : TERNARY

4B3T: 4 bits in, 3 × TERNARY (3-level) output

$3^3 \Rightarrow (3 \text{ choices})^{3 \text{ symbols}} = 27$



$1\% = \frac{1}{100} = 0.01$

$1 \text{ p.u.} = \frac{1}{1} = 1$

$100 \frac{\text{parts}}{\text{million}} = \frac{100}{10^6}$

$1 \text{ ppm} = \frac{1}{10^6} = 10^{-6}$

e.g. 1% of $1 \text{ MHz} = 10^{-2} \times 10^6 = 10^4 = 10 \text{ kHz}$

eg $100\,ppm$ of $1\,MHz = 100 \times 10^{-6} \cdot 10^{6} = 100\,Hz$

Exercise 5: A link operates at 100 Mb/s. What is the bit period? The transmitter and receiver have independent clocks (oscillators) with accuracies of 100ppm. What is the maximum difference between the two clock periods in ppm? In seconds? ← over one bit. The error accumulates over time. How many bits will it take for the accumulated error to equal 10% of the clock period?

$f_b = 100\,Mb/s$ $\quad T_b = \frac{1}{100 \times 10^6} = \frac{1}{10^8} = 10^{-8}$

$= 10\,ns$ $\quad = 1 \times 10^{-8} = 10 \times 10^{-9}$

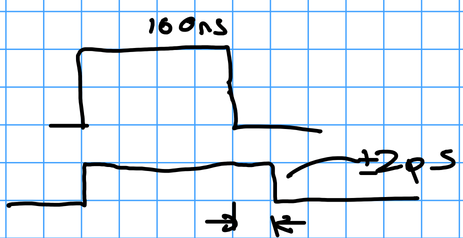
worst case difference is 200 ppm (+100 for one, −100 for other)

in 10 ns worst case difference is $10\,ns \times \pm 200\,ppm = 10 \times 10^{-9} \times 200 \times 10^{-6}$

$= 2000 \times 10^{-15}$

$= 2 \times 10^{-12}$

$= \pm 2\,ps$



10% of 10 ns is 1 ns

$\frac{1\,ns}{2\,ps} = \frac{1 \times 10^{-9}}{2 \times 10^{-12}} = 0.5 \times 10^{+3} = 500$ bits //

Exercise 6: What is the probability of having 100 consecutive 1's in a stream of random bits? How often would this happen at a bit rate of 1 Gb/s? (Hint: 1 Gb/s is about 2^30 bits/second, 1 year has about 2^25 seconds).

2^{30} $\quad 2^{25}$

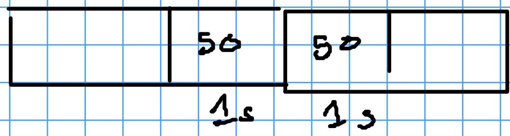
$P(100 \text{ consecutive } 1\text{'s}) = P(1) \cdot P(1) \ldots$

$= \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdots$

$\simeq \left(\frac{1}{2}\right)^{100} = 800 \times 10^{-33} \simeq 1 \times 10^{-30}$

$1\,Gb/s = 2^{30}$.

symbol rate at 100 bits/symbol $= \frac{1\,Gb/s}{100\,bits/sym} = \frac{10^9}{10^2} = 10^7\,\frac{sym}{s}$



$$10^{7} \times 10^{-30} \simeq 10^{-23} \ \frac{\text{symbols}}{s}$$

("bad" symbols)

$$1 \times 10^{9} \ \frac{b}{s} \cdot 1 \times 10^{-30} = \underline{1 \times 10^{-21}} \ \text{events/second}$$

$$2^{24} \simeq 16M$$

$$2^{25} \simeq 32M$$

$$\simeq 3 \times 10^{6}$$

Exercise 7: A data link operates over a distance of 10m operates at 100 kb/s. If the velocity factor of the cable is 0.66, what is the propagation delay in microseconds? What fraction of the bit period does this represent?

$$\frac{1}{100 \text{ kb/s}} = 10^{-5} \text{ s} = 10 \mu s$$

$$T_{prop} = \frac{d}{VF \cdot C} = \frac{10}{0.66 \cdot 3 \times 10^{8}} = 5 \times 10^{-8} = 50 \text{ ns}$$

$$\frac{T_{prop}}{T_{bit}} = \frac{50 \text{ ns}}{10 \mu s} = \frac{50 \times 10^{-9}}{10 \times 10^{-6}} = 5 \times 10^{-3} = 0.5\%$$