APSC 380 : INTRODUCTION TO MICROCOMPUTERS
1997/98 WINTER SESSION TERM 1

Number Systems

This lecture reviews the use of binary and hexadecimal notation.
After this lecture you should be able to: convert an integer value between decimal, binary and hexadecimal representations

Number Systems

We commonly use decimal notation to express the value of a number. However, computer hardware represents numbers using two-valued (binary) values. To be able to deal with the hardware representation of numbers we need to be able to convert between the decimal and binary representation of numbers. Note that the value of a number does not change when we express it in a different notation.

Binary Representation

In the decimal number system each digit position represents a different power of ten. The rightmost digit gives the number of multiples of 10^0 in that number, the next digit gives the multiples of 10^1, and so on. For example, $105 = 5 \times 10^0 + 0 \times 10^1 + 1 \times 10^2 = 5 \times 1 + 0 \times 10 + 1 \times 100$.

Binary numbers work in the same way, but since only two values are possible for each digit, each position represents a different power of two. The rightmost digit gives the number of multiples of 2^0, the next digit gives the multiples of 2^1, and so on. For example, $110 = 0 \times 2^0 + 1 \times 2^1 + 1 \times 2^2 = 0 \times 1 + 1 \times 2 + 1 \times 4 = 6_{\text{(base 10)}}$.

The table below shows the decimal values of powers of 2 for exponents from 0 to 15:

exponent	value
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128
8	256
9	512
10	1024
11	2048
12	4096
13	8192
14	16384
15	32768

To convert from binary to decimal we just need to add up the powers of two corresponding to the bit positions that are '1's.

Exercise: Convert the binary number 1001 0110 to a decimal number.

To convert from decimal to binary it is necessary to find the appropriate combination of powers of two that will add up to the desired decimal number. This is done by repeatedly subtracting the largest possible power of two until the remaining value is zero.

For example, to convert the decimal value 35, we find the largest power of two less than or equal to 35: 32 (2^5). The remainder is $35 - 32 = 3$. The next largest power of that can be subtracted is $2^1 = 2$. Subtracting this value leaves $3 - 2 = 1$. The next possible power of two is $2^0 = 1$. Subtracting this value leaves 0. Therefore the binary value is 10011.

Exercise: Convert the decimal number 86 to a binary number.

Hexadecimal Representation

Binary numbers are too verbose for many purposes so we use often use hexadecimal (base 16) numbers.

Hex numbers are less verbose but also easier to convert to binary. Since the base is 16, we need 16 different digits. We use the digits 0 to 9 and the letters A to F. The following table shows the the 16 hexadecimal digits and the corresponding values in decimal and binary.

decimal	binary	hex
0	0000	0
1	0001	1
2	0010	2
3	0011	3
4	0100	4
5	0101	5
6	0110	6
7	0111	7
8	1000	8
9	1001	9
10	1010	A
11	1011	B
12	1100	C
13	1101	D
14	1110	E
15	1111	F

To convert from binary to hex we group the binary digits into groups of 4 starting from the least significant (rightmost) digit. The we just look up the corresponding hexadecimal value in the table.

Exercise: Convert the binary number 1101000 to hexadecimal.

Hex notation is used because the conversion between hex and binary is much simpler than between decimal and binary. This is becase each hex digit represents exactly 4 bits.

To convert a decimal number to hex you can first convert the number to binary and then group the bits into groups of 4 bits starting from the right. These 4-bit patterns are then easily converted to hex digits.

Similarly, to convert a hex number back to decimal first convert it to a binary number and find the corresponding decimal value.

Exercise: What are the binary and decimal representations of the hexadecimal value 3F?

Hexadecimal Constants in C

We can use hexadecimal notation in C programs by prefixing the constant using the characters ’0x’. The hex digits may be in upper of lower case.