Solutions to Assignment 3

Question 1

With the best possible FEC the error-free throughput of the channel will be equal to the Shanon capacity. For the BSC, the capacity is:

$$C = 1 - (-p \log_2 p - (1-p)\log_2(1-p))$$

The question says $p = 0.01$ so the capacity in bits per channel use is:

$$C = 1 - (-0.01 \log_2 0.01 - (0.99)\log_2(0.99))$$

$$= 1 - (0.066 - -0.014) = 0.92$$

Since the system transmits 1 Mb/s (1 M channel uses per second), the error-free throughout would be limited to 0.920 Mb/s.

Question 2

(a) The channel response extends to 1.3 MHz and the minimum bandwidth is 1.1 MHz so the excess bandwidth parameter is $\alpha = \frac{0.2}{1.1} = 0.182$.

(b) The symbol rate can be transmitted over this channel without ISI is twice the −6 dB bandwidth or $2 \times 1.1 = 2.2$ Msymbols/s.

Question 3

PPP encapsulation requires using a flag character before and after the frame and an escape character before each flag, escape or other special character. If we also XOR each escaped character with 0x20 the result would be:

```
0x7e          <- flag
0x00, 0x27,
0x7d, 0x5e,   <- escaped 0x7e
0x7d, 0x5e,   <- escaped 0x7e
0x7d, 0x5d,   <- escaped 0x7d
0x11
0x7e          <- flag
```

Question 4

(a) A code with a minimum distance of 3 can correct $\lfloor \frac{3-1}{2} \rfloor = 1$ error per codeword.

(b) The probability of a specific bit being received in error is $p = 10^{-3}$. So the probability of a specific bit not being in error is $1-p$. For a 7-bit codeword the probability of a specific one-bit error pattern happening is $p^1(1-p)^6$ but there are 7 possible locations for the error so the probability of any of these happening is $7p(1-p)^6$. The probability of receiving a 7-bit codewords that contains exactly one error is thus $7\times10^{-3}\times(1-10^{-3})^6 \approx 7\times10^{-3}$.