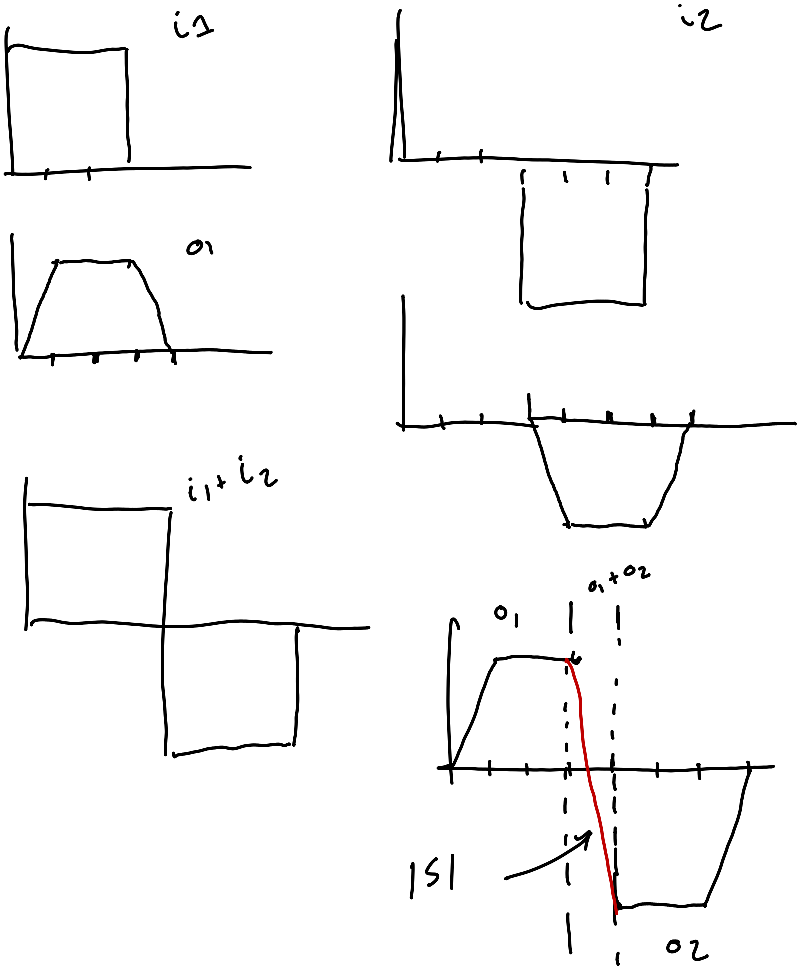
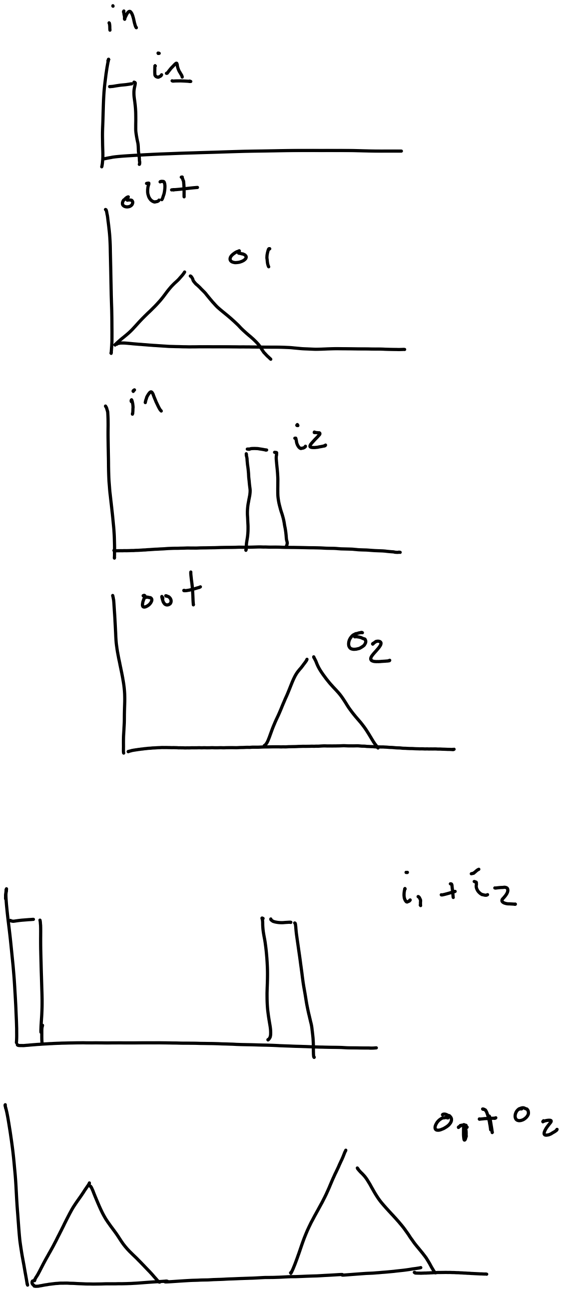
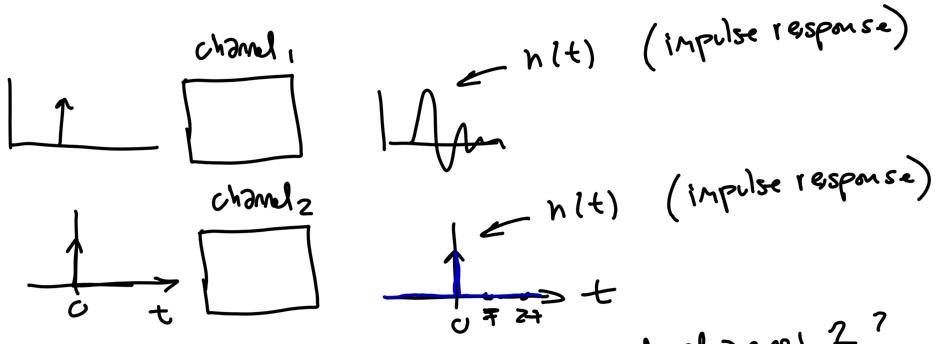
Data Transmission over Bandlimited Channels

Exercise 1: Draw a square pulse of duration T. Draw the pulse after it has passed through a linear low-pass channel that results in rise and fall times of $T/3$. Draw the output for an input pulse of the opposite polarity. Use the principle of superposition to draw the output of the channel for a positive input pulse followed by a negative input pulse.





Exercise 2: What is the impulse response of a channel that does not alter its input? Does this impulse response meet the Nyquist condition? Will it result in ISI?



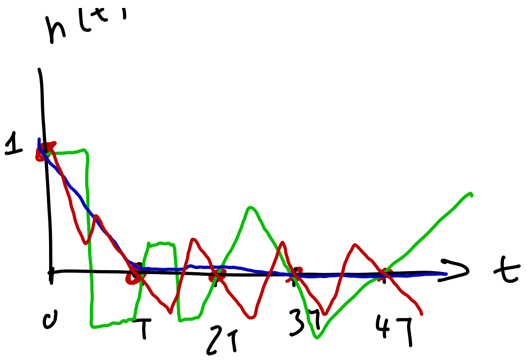
what is the impulse response of channel 2?

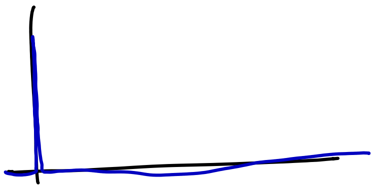
- (an impulse).

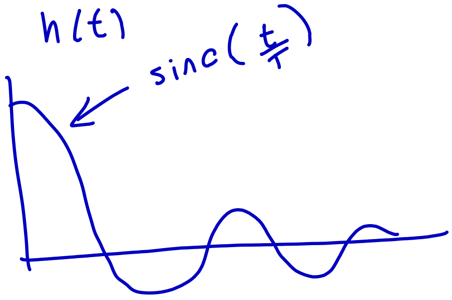
- yes, it is zero at all multiples of T. (no matter what T is).

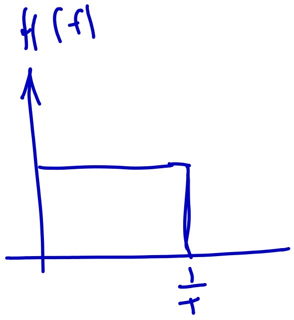
- No, no ISI (regardless of the value of T).

Exercise 3: Draw the impulse response of a channel that meets the Nyquist condition but is composed of straight lines.









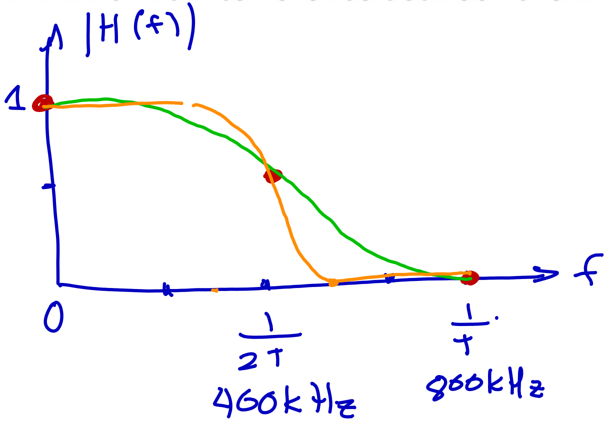
Exercise 4: Draw the magnitude of a raised-cosine transfer function that would allow transmission of impulses at a rate of 800 kHz with no interference between the impulses.

$T = \frac{1}{f} = \frac{1}{800\,kHz}$

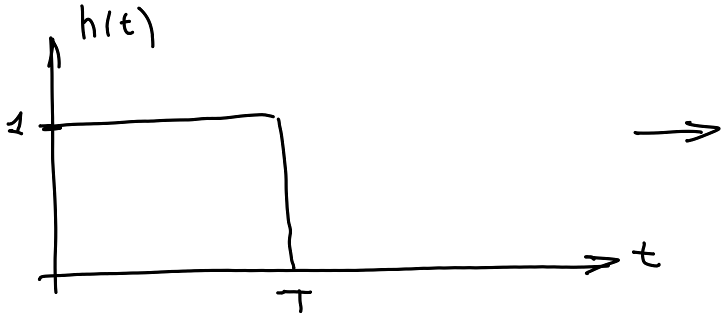
$\frac{1}{T} = 800\,kHz$

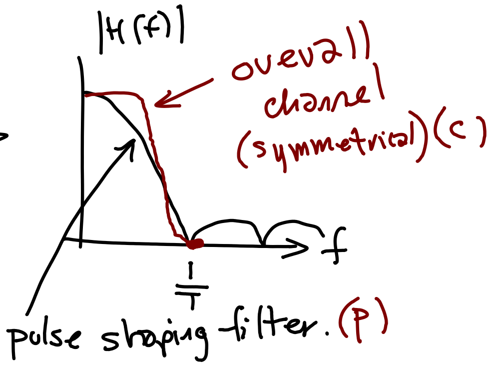
$\frac{1}{2T} = \frac{1}{2} \cdot 800\,kHz$

$= 400\,kHz$



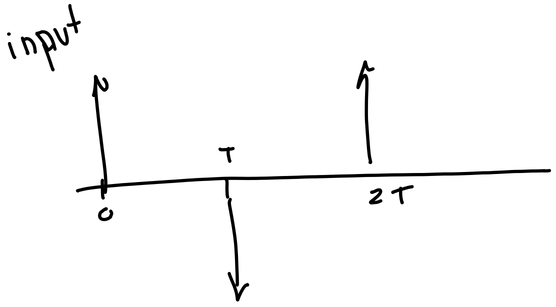
Exercise 5: Draw the impulse response of a filter than converts input impulses to pulses of duration T? Draw the signal after the pulse-shaping filter in the diagram above.

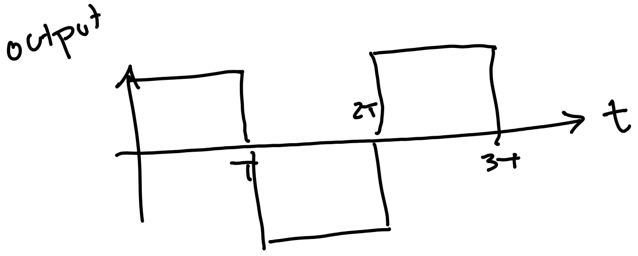


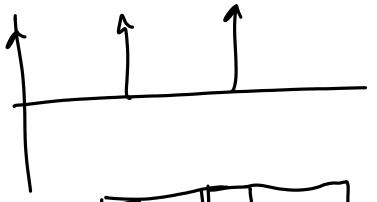


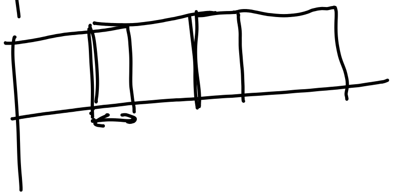
P + channel + equalizer

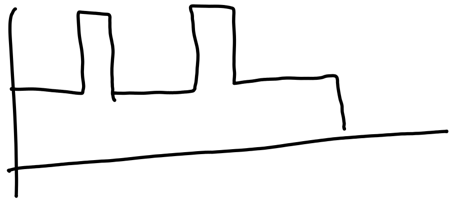
has to meet symmetry conditions.





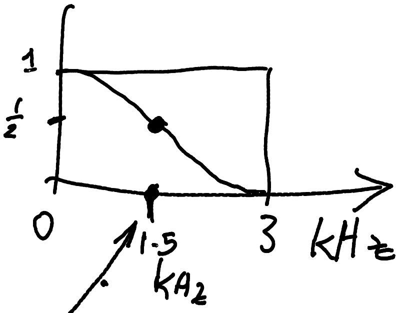






$0 < \alpha < 1$

Exercise 6: A channel has a 3 kHz total bandwidth and meets the Nyquist non-ISI conditions with $\alpha = 1$. How many levels are required to transmit 24 kb/s over this channel using multi-level signalling?



$$\frac{1}{2T} = \frac{1}{2}R$$

$$\frac{1}{2}R = 1.5\ \text{kHz}$$

$$R = 3\ \text{kHz}$$

$$\frac{\text{bit rate} = 24\ \text{kb/s}}{\text{symbol rate} = 3\ \text{k symbols/s}} = 8\ \frac{\text{bits}}{\text{symbol}}$$

need $2^8 = 256$ levels.

Exercise 7: The 802.11g WLAN standard uses OFDM with a sampling rate of 20 MHz, with $N = 64$ and guard interval of $0.8\mu s$. What is the total duration of each OFDM block, including the guard interval? How long is the guard time? — in samples.

N = number of samples / block = 64

$f_s = 20 MHz$

$T_g = 0.8 \mu s.$

total duration = block duration + guard time

64 · sample duration + 0.8μs

$= \frac{64}{20 \times 10^6} + 0.8 = 3.2 + 0.8 = 4 \mu s$

Samples in guard time:

$= 0.8 \times 10^{-6} s \times 20 \times 10^6$ samples/second.

= 16 samples

Exercise 8: What is capacity of a binary channel with a BER of $\frac{1}{8}$ (assuming the same BER for 0's and 1's)?

$p = \frac{1}{8}$ what is C?

$$C = 1 - (-p\log_2 p - (1-p)\log_2(1-p))$$

$$= 1 - \left(-\frac{1}{8}(-3) - \frac{7}{8}(-0.2)\right)$$

$$= 1 - \left(\frac{3}{8} + \frac{1.4}{8}\right) = 1 - \frac{4.4}{8} \approx \frac{3.6}{8}$$

$$\log_2(p) = \log_2\left(\frac{1}{8}\right)$$

$$= \log_2(2^{-3})$$

$$= -3$$

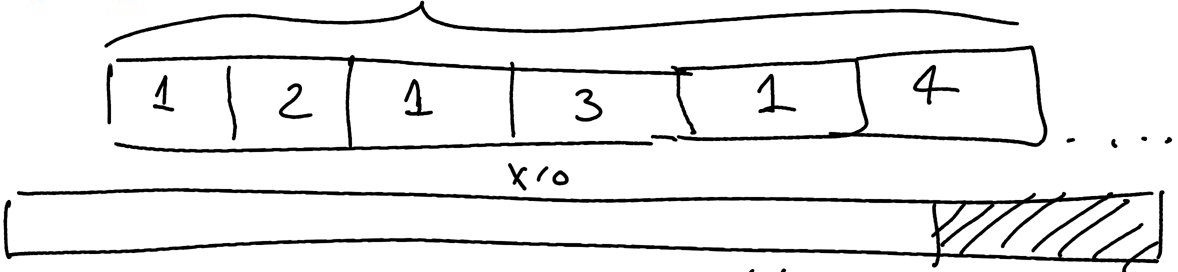
$$\log_2(1-p) = \log_2\left(\frac{7}{8}\right)$$

$$\approx -0.2$$

Question 5

Four users share a communication channel that is divided into 100 μs time slots. One user uses half of the time slots and the other half of the time slots are shared equally between three low-priority users. 96 bits of useful data can be transmitted in each time slot. 10% of the slots contain errors and cannot be used. What throughput, in bits per second, do the three low-priority users see?

10% of the $\frac{\text{slot}}{\text{slots}}$ $\approx$



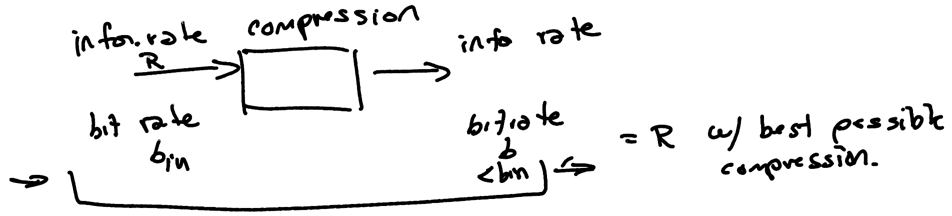
6 X 10 = 60 slots —

		# lost
30	for user 1	3
10	2	1
10	3	1
10	4	1
		6

9 slots of data

in 60 slot times

Exercise 9: Can we use compression to transmit information faster than the (Shannon) capacity of a channel? To transmit data faster than capacity? Explain.



answer: No. compression cannot reduce the information rate $(b_{out} \lesssim R)$

↑ best possible.

→ answer: Yes. Capacity limits information rate not bit (or data) rate.

Exercise 10: What is the channel capacity of a 4 kHz channel with an SNR of 30dB?

$$C = B \log_2 \left(1 + \frac{S}{N}\right)$$

$$= 4 \times 10^3 \underbrace{\log_2 \left(1 + 10^{\frac{30}{10}}\right)}_{}$$

$$\simeq 40 \text{ kb/s}$$

$$\log_2 (\overbrace{1024}^{2^{10}}) = 10$$

Exercise 11: What do the Nyquist no-ISI criteria and the Shannon Capacity Theorem limit? What channel parameters determine these limits?

	Nyquist	Shannon
limits	symbol rate	information rate
based on	-6dB ($\frac{1}{2}$) bandwidth of channel (2x)	Bandwidth S/N } for AWGN channel. P } for BSC