Solutions to Final Exam

Question 1

If four messages have probabilities 0.25, 0.25, 0.125 and 0.375 the the entropy of the source is:

$$H = \sum_i (-\log_2(P_i) \times P_i) \text{ bits/message}$$

which for $P_i = 0.25, 0.25, 0.125$ and 0.375 evalutes to:

$$H = -0.25\log_2(0.25) - 0.25\log_2(0.25) - 0.125\log_2(0.125) - 0.375\log_2(0.375) \approx 1.9 \text{ bits/message}$$

as computed by the following spreadsheet:

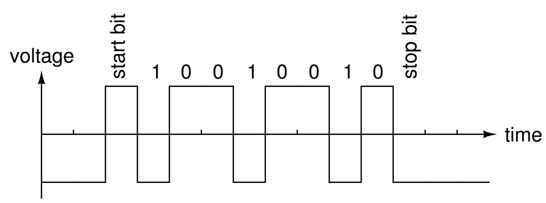
P(i)	-P(i)*log2(Pi)
0.25	0.5
0.25	0.5
0.125	0.375
0.375	0.531
sum=	1.906

Since the message rate is 1000 messages/second (1 ms per message), the information rate of the source is 1906 bits/second.

This information rate is also the bit rate that results when the best possible compression is used.

Question 2

The bits transmitted by the given RS-232 waveform are drawn below:



Since the bits are transmitted from l.s. to m.s. bit, the value transmitted was thus $0100\ 1001_2$ =0x49. This corresponds to the letter 'I'.

Question 3

A low-pass channel with a gain of 0.5 (−6 dB) at a frequency of 0.6 MHz and that is symmetrical about that frequency will be able to pass signals at a symbol rate of twice that, $2 \times 0.6 = 1.2$ MHz without ISI.

If each symbol is a pulse that has one of 8 possible levels, then each symbol can represent $\log_2(8) = 3$ bits and the data rate is $3 \times 1.2 = 3.6$ Mbits/second.

Question 4

The message polynomial $x^5 + x^2 + x + 1$ represents the bit sequence 100111 and the generator polynomial $x^2 + 1$ the bit sequence 101. The long division of the message by the generator polynomial is:

```
        1011
      ------
101  |100111
      101
      ---
       011
       ---
        111
        101
        ---
         101
         101
         ---
          00
```

Since the remainder is zero, the channel did not introduce any errors.

Question 5

The Hamming distance, d, of a code with the two codewords 1001 and 0110 is the only Hamming distance, the number of bits that differ between 1001 and 0110 which is 4 (all the bits differ).

This code could detect $d - 1 = 3$ errors and correct $\lfloor \frac{d-1}{2} \rfloor = \lfloor 1.5 \rfloor = 1$ error.

Since there are 2 codewords, each codeword represents (transmits) $\log_2(2) = 1$ bits.

Question 6

The first six bytes of an Ethernet frame following the SFD are the destination address and the next six are the source address.

In this case the destination address is: `bc:83:85:f9:7d:7c` and the source address is: `00:1d:60:9f:21:94`.

Question 7

(a) The maximum period of a PN sequence generated by a circuit using $m = 12$ flip-flops is $2^m - 1 = 2^{12} - 1 =$4095 bits.

(b) The time and amplitude of noise caused by lightning from storms would be unpredictable so it would be considered a random rather than pseudo-random signal.

(c) A system transmitting data in the form of frames, such as WiFi, is more likely to use an additive scrambler (mistakenly called multiplicative in the exam question) rather than a convolutional scrambler because the frame boundaries can define the start of the scrambling sequence.

Since the question used the wrong terminology, I did not mark this part and the maximum mark for this question was 7.

(d) It would be most appropriate to use go-back-N ARQ for a communication link with a delay that is long relative to the frame duration (such as the 2.5 second round-trip propagation delay to the moon relative to 1 ms frame duration) and a low error rate (every 10 years in the example) because it would provide high throughput (assuming $N > 2500$) with lower complexity that selective-repeat ARQ.

(e) A communication system with devices that have a range of about 3 m would be considered a PAN since this is much less than the typical range of a LAN (about 100 m).