Solutions to Assignment 1

Question 1

The signal must be sampled at more than twice the bandwidth. The bandwidth is 300 Hz so the sampling rate must be at least 600 Hz.

The quantization SNR is about 6 dB per bit[1]. Thus for a 60 dB ratio of signal power to quantization noise power we need $\frac{60\text{ dB}}{6\text{ dB/bit}} = 10$ bits.

Question 2

UTF-16 encodes characters using 16-bit values. The most common characters can be encoded as a single 16-bit value but others must be encoded as two 16-bit values.

Question 3

A TV station transmits a 50 kW signal at 200 MHz. The transmit antenna has a gain of 16 dB. At what distance from the transmitter will a receiver with an omnidirectional antenna (gain=0 dB) receive a power of 1 μW?

The received power is given by the Friis equation as:

$$P_R = P_T G_T G_R \left(\frac{\lambda}{4\pi d}\right)^2$$

In this question $P_T = 50 \times 10^3$ W, $f = 200 \times 10^6$ so that $\lambda = \frac{c}{f} = \frac{300\times10^6}{200\times10^6} = 1.5$ m, $G_T = 10^{16/10} \approx 40$, $G_R = 1$ (0 dB) and $P_R = 1 \times 10^{-6}$. Solving for d:

$$\frac{P_R}{P_T G_T G_R} = \left(\frac{\lambda}{4\pi d}\right)^2$$

$$\sqrt{\frac{P_T G_T G_R}{P_R}} = \frac{4\pi d}{\lambda}$$

$$d = \frac{\lambda}{4\pi}\sqrt{\frac{P_T G_T G_R}{P_R}}$$

$$d = \frac{1.5}{4\pi}\sqrt{\frac{50 \times 10^3 \cdot 40 \cdot 1}{1 \times 10^{-6}}} = 169 \text{ km}$$

But to achieve a line of sight (LOS) path over this distance would require a very high (560 m high) towers.

Question 4

What is the characteristic impedance of a pair of 12-gauge wires running next to each other? Assume each wire's insulation is 1 mm thick.

The diameter D of 12-gauge wire is:

$$d \approx 8 \times 2^{-\frac{12}{6}} = 2 \text{ mm}$$

and if the insulation is 1 mm thick, the spacing S between the centers of the wires is $S = 1+1+1+1 = 4$ mm and the characteristic impedance is:

$$Z_0 \approx \frac{120}{\sqrt{\varepsilon_r}} \ln\left(\frac{2S}{D}\right)$$

and assuming a dielectric constant of $\varepsilon_r = 2.2$ (not given in the question, another assumption would also be correct),

$$Z_0 \approx \frac{120}{\sqrt{2.2}} \ln\left(\frac{2 \times 4}{2}\right) \approx 112\ \Omega$$

if ε_r has been assumed 1 (air dielectric/insulation) then $Z_0 \approx 166\ \Omega$.

[1] This is only true if there is no clipping and the signal is uniformly distributed over the quantization range. For example, for a sinusoid the quantization SNR is about 1.8 dB higher assuming the input extends over the quantization range.