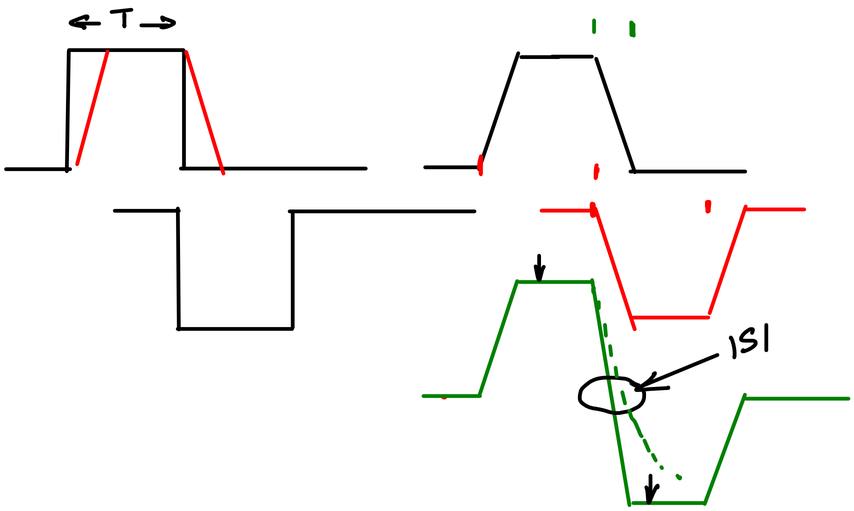
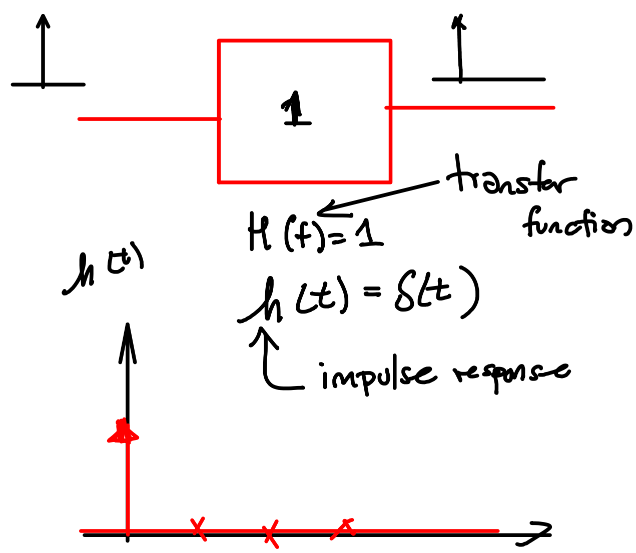
Lecture 7 - Data Transmission over Bandlimited Channels

Exercise 1: Draw a square pulse of duration T. Draw the pulse after it has passed through a linear low-pass channel that results in rise and fall times of $T/3$. Draw the output for an input pulse of the opposite polarity. Use the principle of superposition to draw the output of the channel for a positive input pulse followed by a negative input pulse.



Exercise 2: What is the impulse response of a channel that does not alter its input? Does this impulse response meet the Nyquist condition? Will it result in ISI?

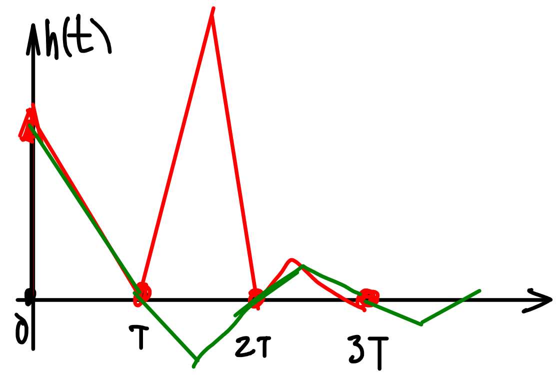


$\delta(t)$

Yes, in fact, it's zero everywhere, not just at T, 2T, 3T, ...

no ISI

Exercise 3: Draw the impulse response of a channel that meets the Nyquist condition but is composed of straight lines.

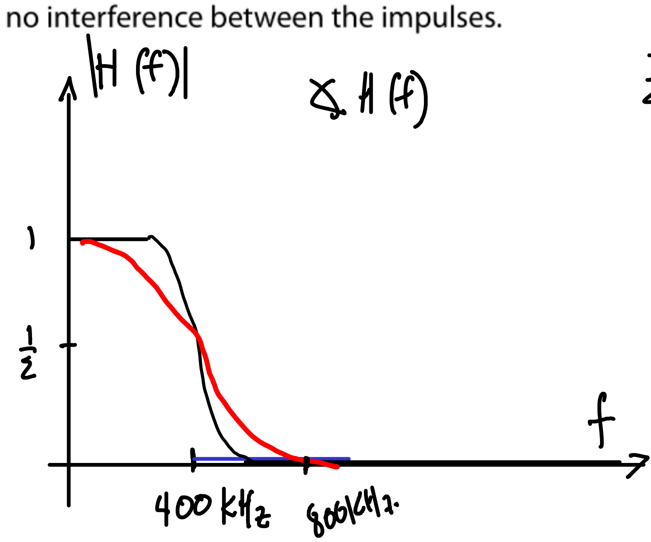


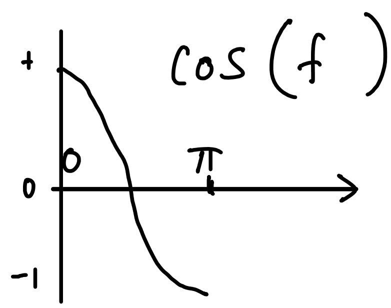
$f = \frac{1}{T}$

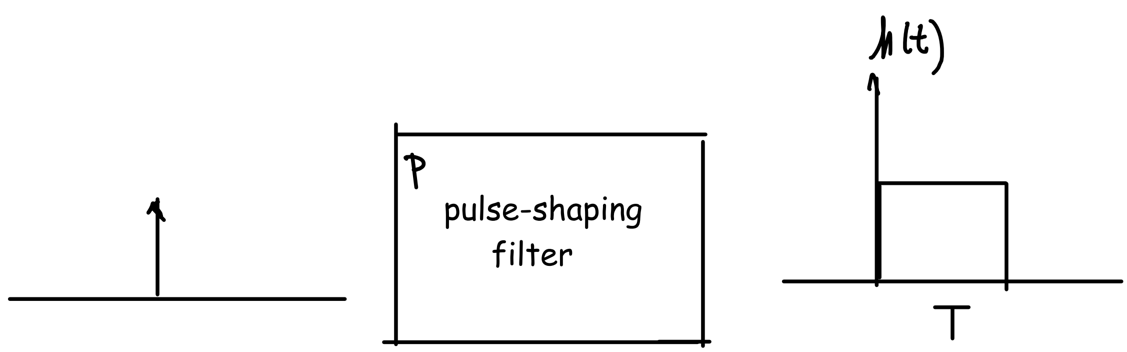
Exercise 4: Draw the magnitude of a raised-cosine transfer function that would allow transmission of impulses at a rate of

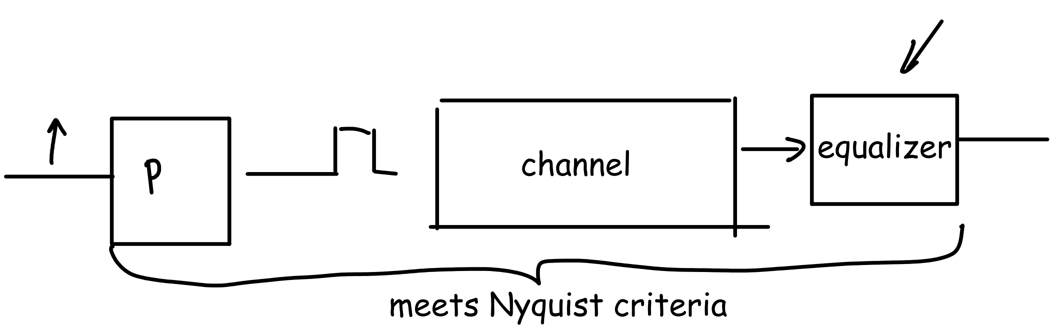
$f =$ 800 kHz with no interference between the impulses.

$\frac{1}{2T} = \frac{1}{2} \cdot \frac{1}{T} = \frac{1}{2} f$









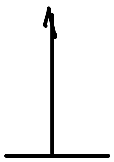
Exercise 5: Draw the impulse response of a filter than converts input impulses to pulses of duration T? What is the shape of the frequency response of this filter? *Hint: the Fourier transform of a pulse of duration T is* $\frac{\sin(\pi f T)}{\pi f}$. What is the "bandwidth" of this filter – when is it first zero? How does this compare to the "bandwidth" of the raised-cosine filter above?

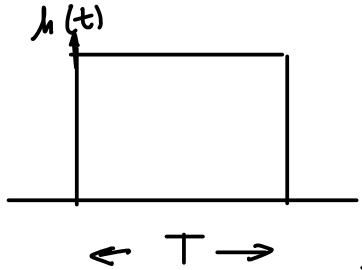
$\pi f T = 0, \pi, 2\pi \ldots \; -\pi, -2\pi$

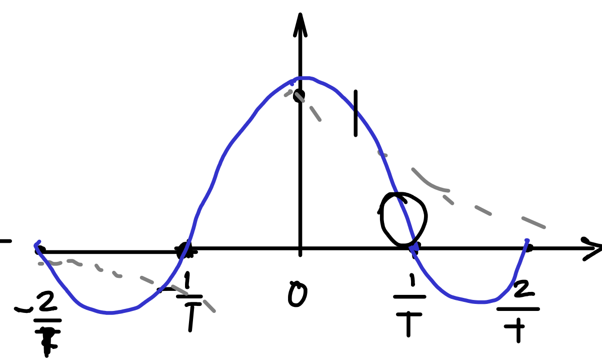
when $f =$

$\cancel{\pi} f T = \cancel{\pi}$

$f = \frac{1}{T}$



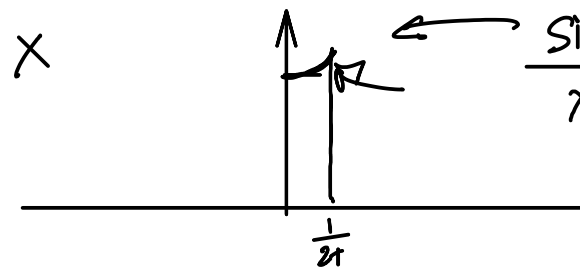




DSP → symbol

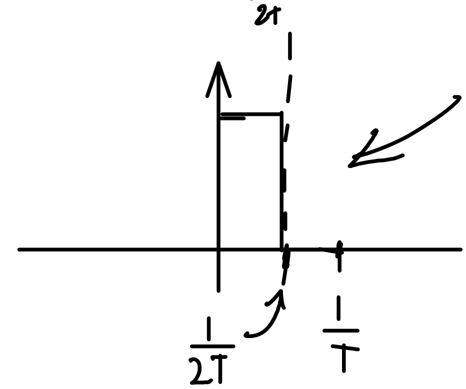
3525 → samples

rate	b/w
2	1



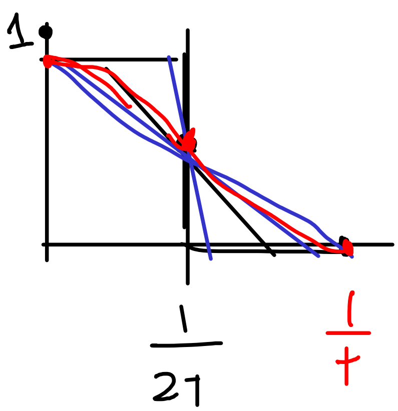
$\frac{\sin x}{x}$ correction

$=$



to get this

Exercise 6: What is the possible range of values of α?



$0 < \alpha \leq 1$

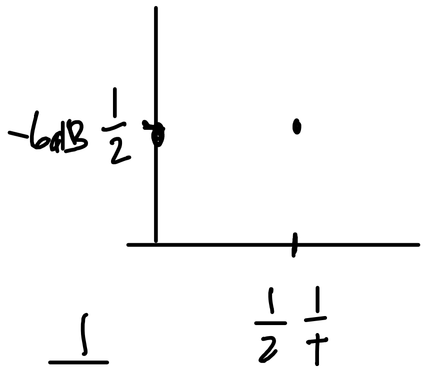
$f \quad \times \quad f \quad \Rightarrow RC$

$$\underbrace{\to \boxed{\frac{1}{\sqrt{x}}} \to \boxed{\frac{1}{\sqrt{x}}}}_{RC} \to \frac{1}{x}$$

$$\frac{1}{x} \cdot \frac{1}{x} = \frac{1}{3}$$

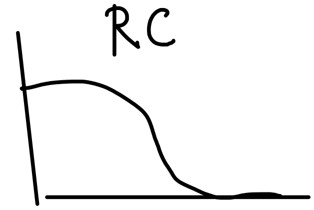
$$\frac{1}{x^2} = \frac{1}{3}$$

$$\frac{1}{x} = \frac{1}{\sqrt{3}}$$



$\frac{1}{\sqrt{2}}$

RC



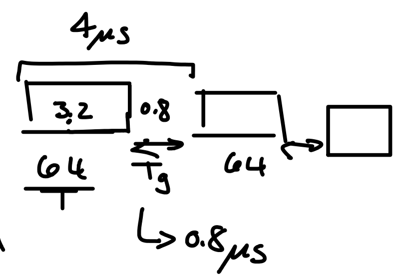
$f^2 \Rightarrow RC$

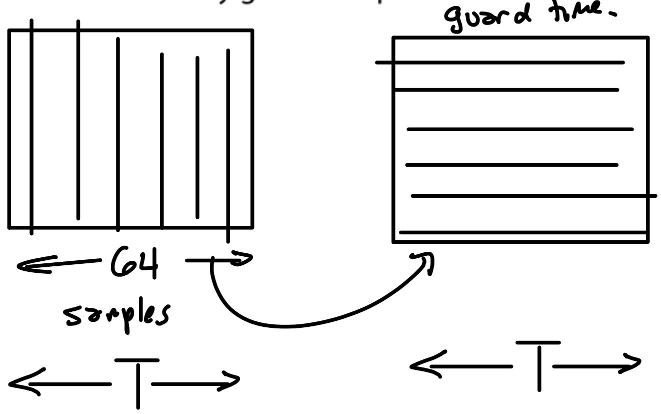
RRC

$$\overset{f}{\boxed{\frac{1}{\sqrt{2}}}} \cdot \overset{f}{\boxed{\frac{1}{\sqrt{2}}}} \to \frac{1}{2}$$

-3 dB $+ -3$ dB $\to -6$dB

Exercise 7: The 802.11g WLAN standard uses OFDM with a sampling rate of 20 MHz, with $N = 64$ and guard interval of $0.8\mu s$. What is the total duration of each OFDM block, including the guard interval? How many ~~guard~~ samples ~~are used?~~ long is the guard time.





64 subcarriers

$$f = \frac{N}{T}$$

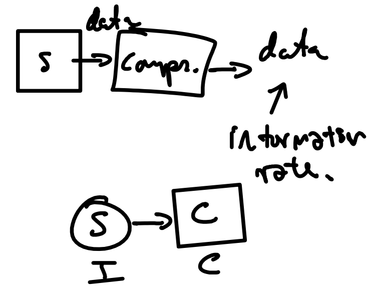
16 guard samples

$f_{sample} = 20$ MHz

$$T = 3.2\ \mu s = 64 \cdot \frac{1}{20 \times 10^6}$$

Exercise 8: Can we use compression to transmit data faster than the Shannon capacity? Explain.

information

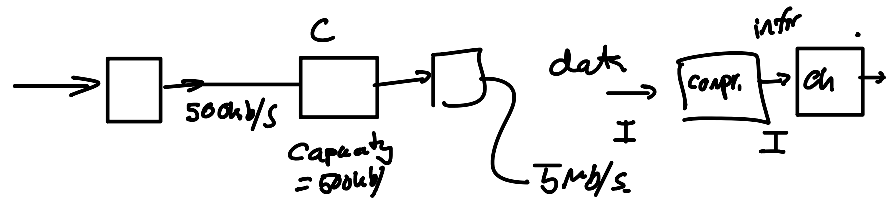


NO. - compression doesn't affect the information rate

(information) - capacity limits the information rate

YES - e.g.

(data) 5Mb/s



Exercise 9: What is the channel capacity of a 4 kHz channel with an SNR of 30dB?

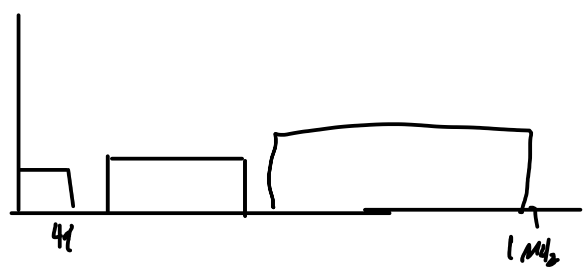
V.28
V.56

$$C = B \log_2\left(1+\frac{S}{N}\right) = 4 \times 10^3 \log_2\left(1 + 10^{\frac{30}{10}}\right)$$

$$= 4 \times 10^3 \log_2(1001) \approx \underline{\underline{40\,kb/s.}}$$

Q. SNR = $6 \cdot B$, $B = 8$

$= 6 \times 8 = \underline{\underline{48\,dB}}$



Exercise 10: What are some differences between the signalling rate limit imposed by the Nyquist no-ISI criteria and the Shannon Capacity Theorem? For example, what do they limit and what parameters determine these limits?

	Nyquist	Shanon Capacity
limits	symbol rate	information rate
parameter	impulse response band width (-6dB)	$\underline{S/N}$, $\underline{B}$.