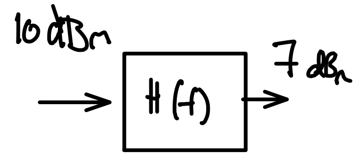
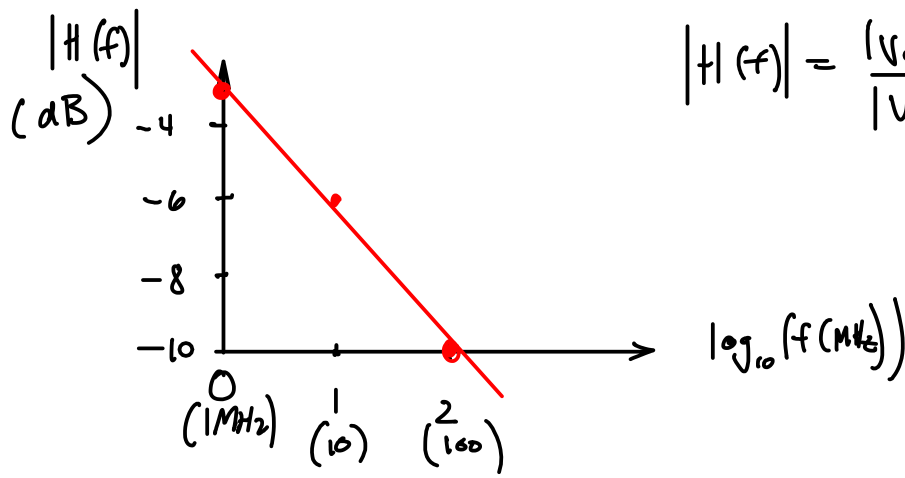
Lecture 4 - Channel Characteristics and Impairments

Exercise 1: A 10 dBm signal is applied to one end of a 50 ohm co-ax cable at frequencies of 1, 10 and 100 MHz. At the other end you measure voltages of 7, 4 and 0 dBm respectively. Plot the amplitude of the transfer function of the channel formed by this cable. Show dB on the vertical axis and log of frequency on the horizontal axis.





$$|H(f)| = \frac{|V_{out}|}{|V_{in}|} = \left|\frac{V_{out}}{V_{in}}\right|$$

$-3dB$: power or voltage? neither and both:

$$\frac{P_{OUT}}{P_{IN}} = \frac{10^{\frac{7}{10}}}{10^{\frac{10}{10}}} = \frac{5}{10} = \frac{1}{2} \qquad 10\log\left(\frac{1}{2}\right) = -3$$

$$\frac{V_{OUT}}{V_{in}} = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$$

$$G(dB) \leftarrow 10\log\left(\frac{P_{OUT}}{P_{IN}}\right) \text{ or } 20\log\left(\frac{V_{OUT}}{V_{IN}}\right)$$

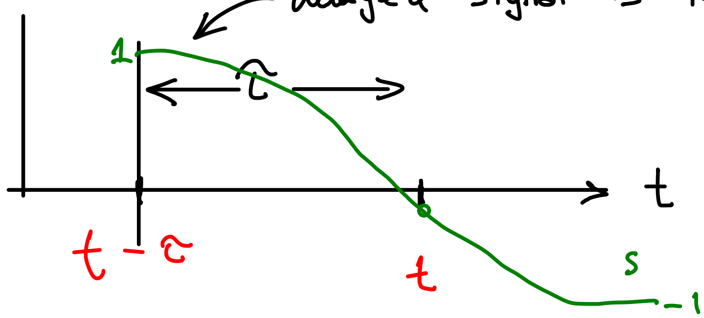
} same answer in dB

Exercise 2: How much *power* would a signal transmitted at the edge of the 3 dB bandwidth passband have compared to the the power it would have if transmitted at the frequency with the lowest loss? What would be the ratio of the *voltages*? What if the bandwidth was defined as the 6 dB bandwidth?

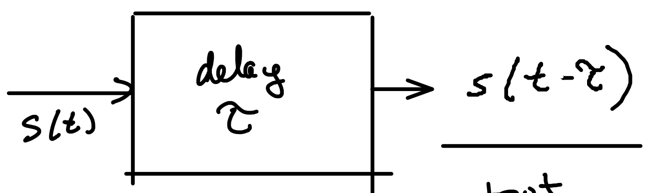
- $-3\,dB$ or $\frac{1}{2}$ power at $-3dB$ frequency.
- $\frac{P_{OUT}}{P_{IN}} = \frac{1}{2} = \frac{V_{OUT}^2}{V_{IN}^2}$ $\qquad \frac{V_{OUT}}{V_{IN}} = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$
- $-6\,dB = \frac{1}{4}$ power $= \frac{1}{2}$ voltage

Exercise 3: A 100m transmission line has a velocity factor of 0.66. Plot the phase response of the cable over the frequency range 0 to 6 MHz.

delayed signal is the one with the earlier value.



$s(t-\tau)$ OR $s(t+\tau)$? — which is the delayed signal?



input: $\cos(2\pi f t)$ $\qquad \theta_{IN} = 2\pi f t$

output: $\cos(2\pi f(t-\tau))$ $\qquad \theta_{OUT} = 2\pi f(t\tau)$

$\angle H(f) = \theta_{OUT} - \theta_{IN} = 2\pi f(t-\tau) - 2\pi f t$

$\measuredangle H(f) = -2\pi f \tau$

$-\pi \times 10^{-6} f$

$\tau = \frac{d}{v} = \frac{100}{3\times10^{8} \cdot 0.66} = 50\times10^{-8}$

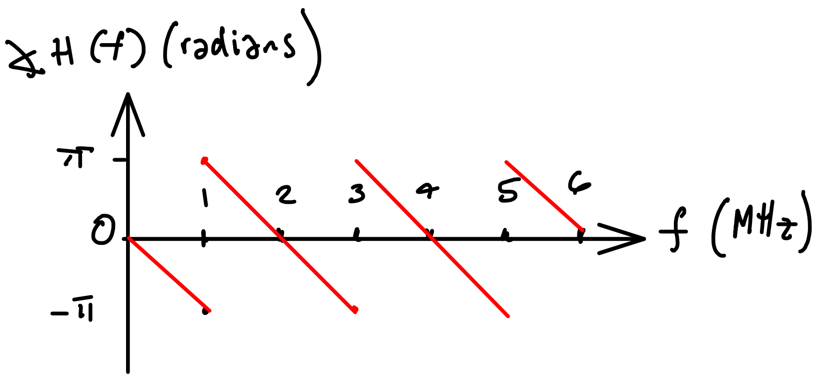
$= 0.5 \mu s.$

at $f = 0$ M → 0 radians

$f = 1$ → π radians

$f = 2$

f (MHz)	$\measuredangle H(f)$ (radians)
0	0
1	$-\pi = \pi$
2	$-2\pi = 0$
3	$-3\pi = -\pi = \pi$
⋮	
6	$-6\pi = 0$



Exercise 4: A telephone line is being used to transmit symbols at a rate of 300 symbols/second using frequencies between 800 and 1200 Hz. If the group delay ~~must~~ variation must be less than 10% of the symbol period, what is the maximum allowable variation in group delay over this frequency range?

symbol period $= \frac{1}{300} = 3.3$ms

maximum group delay variation $= 10\% \cdot 3.3$ms $= \underline{\underline{333 \mu s.}}$

Exercise 5: The input to a non-ideal amplifier is the sum of two sine waves at frequencies of 1 and 1.2 MHz. What are the frequencies of the harmonics of these frequencies? What are the frequencies of the positive third-order IMD products?

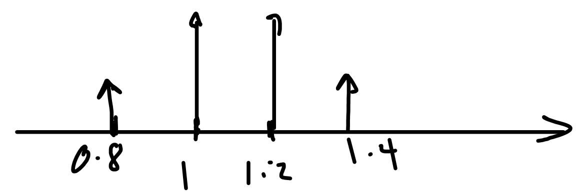
harmonics: of 1: 2, 3, 4, ...
of 1.2: 2.4, 3.6, 4.8, ...

$(n, m) = (1, 2)$ or $(2, 1)$ — only possible values for third-order products

$f_{IM} = \pm n f_1 \pm m f_2$

$f_1 = 1,\ f_2 = 1.2$

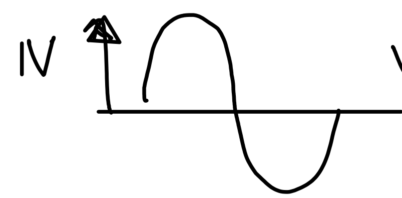
n	m	f_{IM}
+1	+2	3.4
+1	−2	−ve
−1	+2	1.4 * $(-1 + 2.4)$
−1	−2	−ve
+2	+1	3.2
+2	−1	0.8 *
−2	+1	−ve
−2	−1	−ve



* – these are close to the input frequencies. & hard to filter out

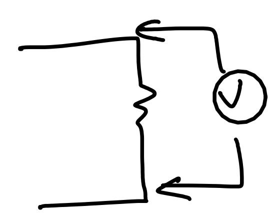
Exercise 6: A sinusoidal signal is being transmitted over a noisy telephone channel. The voltage of the signal is measured with an oscilloscope and is found to have a peak voltage of 1V.

Nearby machinery is adding noise onto the line. The voltage of this noise signal is measured with an RMS voltmeter as 100mVrms. The characteristic impedance of the line is 600Ω and it is terminated with that impedance. Why was an RMS voltmeter used? What is the signal power? What is the noise power? What is the SNR?



A = 1V

$V_{rms} = \frac{1V}{\sqrt{2}}$



– why RMS → not a sinewave

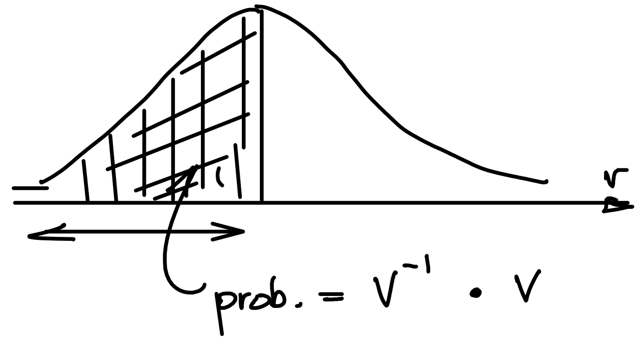
– $S = \frac{V^2}{R} = \frac{\left(\frac{1}{\sqrt{2}}\right)^2}{600} \approx 1mW$ 833μW

$N = \frac{V^2}{R} = \frac{(0.1)^2}{600} = 10\mu W$ 16μW

$SNR = \frac{S}{N} \approx 52$ (17 dB)

Exercise 7: What are the units of t?

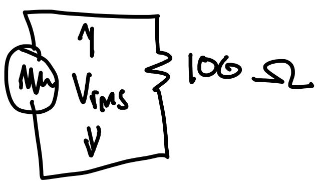
t (normalized threshold) has no units (%, p.u.)



Exercise 8: The output of a noise source has a Gaussian (normally) distributed output voltage. The (rms) output power is 20mW and the output impedance is 100Ω. What fraction of the time does the output voltage exceed 300mV? Hint: the variance (σ^2) of a signal is the same as the square of its RMS voltage.

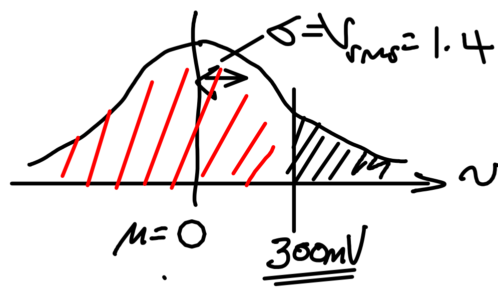
σ^2 = power

σ = voltage (rms)



$P_{rms} = 20mW$

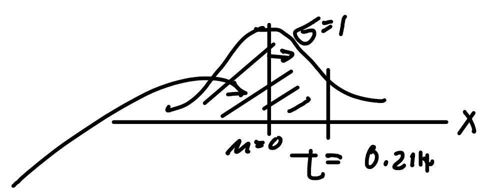
Vrms



$P = \frac{V_{rms}^2}{R}$ assume $\bar{V} = 0$ $(\mu = 0)$

$0.02 = \frac{V_{rms}^2}{100}$ $2 = V_{rms}^2$ $V_{rms} \simeq 1.4$

$t = \frac{0.300 - 0}{1.4} = 0.214$



① $P(0.214) = 0.58$ (calculator)

② $P(0.2) \simeq 0.57$ (graph)

A=

③ $\frac{1}{1 + e^{-1.7t}} = 0.59$ (Logistic approx).

$P(v > 300mV) = P(x > t) = 1 - P(x < t) = 1 - 0.58 = 0.42$

42%