Solutions to Assignment 2

Question 1

(a) If the symbol duration is 0.25 μs the symbol rate is $\frac{1}{0.25\,\mu\text{s}} = 4$ MHz. Since the bit rate is 16 Mb/s there must be $16/4 = 4$ bits/symbol. For multi-level signalling this requires $2^4 = 16$ levels.

(b) There must be one decision threshold between each level so $16 - 1 = 15$ threshold are required.

(c) If the peak-to-peak voltage of the received signal is 2 Vpp and the levels are equally spaced the spacing between levels is $\frac{2}{16-1} = 133$ mV.

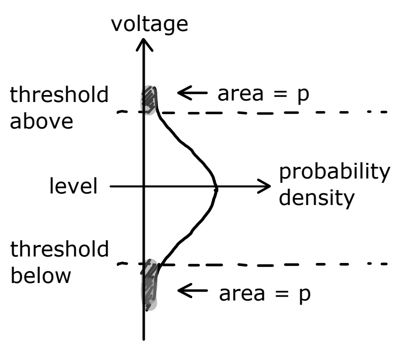
The noise margin is the noise voltage required to cause an error. An error is caused when the signal plus noise crosses a decision threshold. This mean that the noise margin is half the decision threshold spacing or $\frac{1}{2} \times \frac{2}{16-1} = \frac{1}{15} = 66.7$ mV.

(d) (i) For all except the highest and lowest levels: noise causes an error if its *absolute value* is greater than the noise threshold (see diagram below).

(ii) For the lowest level: noise causes an error if it is larger than the noise margin.

(iii) For the highest level: noise causes an error if it is less is less than the negative of the noise margin.

The probability that zero-mean Gaussian noise with a standard deviation $\sigma = 50$ mV is less than -67 mV is $p = P(\frac{-67-0}{50}) = P(-1.33) \approx 9\%$. And from the symmetry of the problem this is also the probability of the noise being greater than 66.7 mV:



although this could also be computed as $p = 1 - P(\frac{67-0}{50}) \approx 9\%$.

For the 14 levels between the highest and lowest levels an error will happen if either threshold is crossed so we sum up the two areas (probabilities) and the symbol error rate if we transmit these levels will be $2p$. For the uppermost and lowest levels the error rate is only p.

If each level has a probability $\frac{1}{16}$ then the average symbol error rate is $\frac{14}{16}2p + \frac{2}{16}p = \frac{30}{16}p \approx 17\%$ However, we can approximate this as $2p \approx 18\%$.

Question 2

A signal source has an open-circuit voltage of 2 Vrms and a purely resistive source impedance of 100 ohms.

(a) (i) the current will be determined by the open-circuit voltage ($V = 2\angle 0$) divided by the total impedance. The total impedance is the sum of the source and load impedances ($Z = 200 + j100 = \sqrt{200^2 + 100^2}\angle \tan^{-1}\left(\frac{200}{100}\right) \approx 223\angle 26.6°\ \Omega$). The current is thus $V/Z \approx \frac{2}{223}\angle(0 - 26.6) = 9\text{ mA}\angle -27°$.

(ii) The power dissipated in the load is $I^2R \approx 8$ mW.

(b) (i) If the source impedance is changed to the complex conjugate of the load impedance, $100 - j100\Omega$, the total impedance changes to 200 Ω (resistive) and the currrent is $2/200 = 10\text{ mA}\angle 0$.

(ii) The power dissipated in the load is now $I^2R = 10$ mW. By setting the source impedance equal to the complex conjugate of the load impedance we have maximized the power delivered to the load.

(c) The reactance of a capacitor is $X_C = \frac{1}{j2\pi fC}$. Solving for C at $f = 100 \times 10^6$ gives $C = \frac{1}{2\pi \times 100 \times 10^6 \times 100} \approx 32 \times 10^{-12} = 16$ pF.

Question 3

According to Wikipedia, 24-gauge wire has a resistance of 0.084 ohms/meter. Thus each 100 m conductor will have a resistance of 8.4 ohms. In both alternatives (A and B) power is supplied over two of these conductors in series (for supply and return) which doubles the resistance but each direction of current flow uses one pair which halves the resistance. So the total loop resistance will be 8.4 ohms.

(a) At a current of 500 mA the voltage drop in the line is $IR = 0.5 \times 8.4 = 4.2$ V so the voltage at the load will be $48 - 4.2 = 43.8$ V. The power supplied by the source is $VI = 48 \times 0.5 = 24$ W, the power consumed by the load is $VI = 43.8 \times 0.5 = 21.9$ W and the power consumed by the cable is $I^2R = (0.5)^2 \times 8.4 = 2.1$ W.

(b) The pinouts for the Bel-Fulse 0813-1X1T-57-F Modular RJ-45 jack with magnetics allow either PoE Alternative A (using pins VC45 and VC78) or PoE Alternative B (using pins VC12 and VC36). The DC line current rating is 350 mA so this part could not be designed into equipment that continously drew or supplied more than 350 mA over PoE.

The high-voltage ("Hi-Pot") isolation rating is given in the datasheet as 2250 VDC.

Question 4

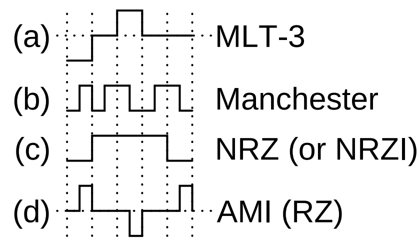
A 48-bit poll or response message sent at 100 kb/s takes $\frac{48}{100\times10^3} = 480\ \mu$s.

(a) We must send one poll and receive the (short) response before polling that station again. If there are 32 slaves each station will be polled once every $32 \times 2 \times 480\ \mu\text{s} \approx 31$ ms.

(b) If each of the slaves transmits a response containing 32 bytes of payload in addition to the header, the response will now take $\frac{8\times32}{100\times10^3} = 2.56$ ms and the time between polls will be $32 \times (480 \times 10^{-6} + 2.56 \times 10^{-3}) \approx 97.3$ ms.

Question 5

The line codes in this example can be distinguished by the number of levels used (2 or 3), by the baud duration relative to the bit duration (one or two transitions per bit), and any relationship between the first and second parts of each bit interval (alternating for Manchester, returning to zero for RZ line codes):



Question 6

A wireless OFDM system is using $N = 256$ and a sampling rate of 5 MHz will have a symbol duration of $T_{symbol} = N \times f_{sampling}$ and a subcarrier spacing of $f_{sampling}/N = 5 \times 10^6/256 \approx 19.5$ kHz.

The symbol duration (not including the guard time) will be $N \times T_{sampling} = N/f_{sampling} = 51.2\ \mu$s.

The time between samples is simply $T_{sampling} =$ 200ns but the time between *symbols*[1] (including the guard time) is $T_{symbol} + T_{guard} = 51.2 + 2 = 53.2\ \mu$s.

Question 7

The (Shannon) capacity of a channel with bandwidth $B = 5$ MHz and an SNR of $\frac{S}{N} = 10^{26/10} \approx 400$ is $C = B \log_2 \left(1 + \frac{S}{N}\right) = 5 \times 10^6 \log_2(1 + 400) = 5 \times 10^6 \log_{10}(1 + 400)/\log_{10}(2) \approx 43$ Mb/s.

[1] What I meant to ask.