Lecture 9 - Error Detection and Correction

Exercise 1: Compute the modulo-4 checksum, C, of a frame with byte values 3, 1, and 2. What values would be transmitted in the packet? What would be the value of the checksum at the receiver if there were no errors? Determine the checksum if the received frame was: 3, 1, 1, C? 3, 1, 2, 0, C? 1, 2, 3, C?

$$C = \left(3 + 1 + 2\right) \text{ modulo } 4 = 6 \text{ modulo } 4 = 2$$

transmit   3, 1, 2, 2

if no errors,   check sum $= 3+1+2+2 \mod 4 = 0$

if   3, 1, 1, 2 $\Rightarrow$ 7 mod 4 $= 3$

   3, 1, 2, 0, 2 $\Rightarrow$ 8 mod 4 $= 0$ $\rightarrow$ insertion of 0 not detected!

   1, 2, 3, 2 $\Rightarrow$ 8 mod 4 $= 0$ $\rightarrow$ re-ordering not detected!

Exercise 2: What is a modulo-2 sum? What is the modulo-2 sum of 1, 0 and 1? What is the modulo-2 sum if the number of 1's is an even number?

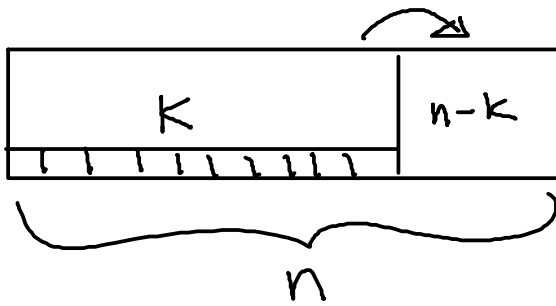modulo-2 sum is remainder after dividing the sum by 2.

$1 + 0 + 1 = 2$; $2 \mod 2 = 0$

even numbers are divisible by 2 $\Rightarrow$ modulo2 $= 0$

Exercise 3: How many possible code words are there for an (n, k) code? How many possible parity bit patterns are possible for each code word?

$$1\ 2\ 3$$
$$2 \times 2 \times 2$$
$$2^3$$

$$\#\ possible\ codewords = 2^k$$

only one possible set of n-k parity bits for each K-data-bit word.

Exercise 4: What is the Hamming distance between the codewords 11100 and 11011? What is the minimum distance of a code with the four codewords 0111, 1011, 1101, 1110?

$$1\ 1\ 1\ 0\ 0$$
$$\oplus\ 1\ 1\ 0\ 1\ 1$$
$$\rule{3cm}{0.4pt}$$
$$0+0+1+1+1 = 3 \leftarrow \text{Hamming distance}$$

$$3 + 2 + 1$$
$$= \underline{6.}$$

↙ number of comparisons for 4 codewords

	0111	1011	1101	1110
0111	0			
1011		0		
1101		②	0	
1110			②	0

(with markings: 0111 column: 0; 1011 column: ②, 0; 1101 column: 1010→②, 0110→②, 0; 1110 column: 1001→②, 0101→②, 0011→②)

$$D_{min} = min\left(2, 2, 2, 2, 2, 2\right) = 2.$$

Larger D_{min} = better.

Exercise 5: A block code has two valid codewords, 101 and 010. The receiver receives the codeword 110. What is the Hamming distance between the received codeword and each of the valid codewords? What codeword should the receiver decide was sent? What bit was most likely in error? Is it possible that the other codeword was sent?

code: 101 , 010   received __110__

$d = 3$

$D_{min} = 3$

101 $\Rightarrow$ distance 2

010 $\Rightarrow$ distance 1 $\Leftarrow$ Choose this as most likely to have been sent.

this was probably the error

Exercise 6: What is the minimum distance for the code in the previous exercise? How many errors can be detected if you use this code? How many can be corrected? What are n, k, and the code rate (k/n)?

$d = D_{min} = 3$

$d - 1 = 2$ errors can be detected (always)

$\left\lfloor \dfrac{d-1}{2} \right\rfloor = \left\lfloor \dfrac{2}{2} \right\rfloor = 1$ error can be corrected (always)

certain received codewords with more errors may be detected or corrected but this is not certain.

Exercise 7: Assume 1000-bit frames are being transmitted at 1 Mb/s. What is the throughput if there are no errors? Now assume errors introduced by the channel require a frame to be retransmitted, on average, 9 times before it is correctly received. What is the throughput now?

Assume we now use a rate-1/2 FEC code. How many bits must be transmitted in each frame? What is the throughput if there are no errors? Now assume the channel introduces errors again but that with FEC coding the receiver corrects most of the errors and, on average, the frame only needs to be transmitted 1.1 times before it is correctly received. What is the new throughput?

When is it worthwhile to use FEC? What other advantage might the use of FEC provide?

$$1 \text{ Mb/s}.$$

$$\frac{9 \times 1000}{10^6}$$

$$\frac{1000}{\frac{9 \times 1000}{10^6}} = \frac{10^6}{9}$$
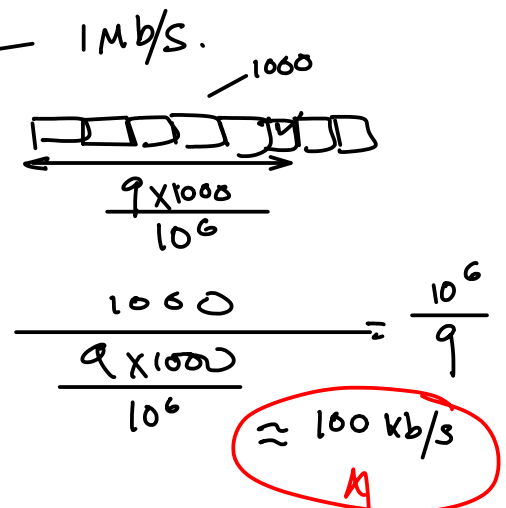
$$\approx 100 \text{ kb/s}$$

$$\frac{k}{n} = \frac{1}{2}$$

$n = 1000$

$k = 500$ data bits/frame.

$$\text{throughput} = \frac{500 \text{ bits}}{\frac{1000 \text{ bits}}{10^6}} = 500 \text{ kb/s}.$$

on "bad" channel :

$$\frac{500}{1.1 \cdot \frac{1000}{10^6}} = 450 \text{ kb/s}$$

Exercise 8: What are the units of Energy? Power? Bit Period? How can we compute the energy transmitted during one bit period from the transmit power and bit duration?

Energy : Joules

Power : Watts

Period : Seconds.

$$\text{Power} = \frac{\text{Energy}}{\text{Time}}$$

$$\text{Eng} = \text{Power} \cdot \text{Time}$$

Exercise 9: A system needs to operate at an error rate of 10^{-3}. Without FEC it is necessary to transmit at 1W at a rate of 1 Mb/s. When a rate-1/2 code is used together with a data rate of 2 Mb/s the power required to achieve the target BER decreases to 500mW. What is the channel bit rate in each case? What is the information rate in each case? What is E_b in each case? What is the coding gain?

	Channel bitrate	bits/s information rate	W. Tx Pow.	J/bit. E_b
w/o FEC	1 Mb/s	1 Mb/s	1 W	$1 \mu J/bit$
w/ FEC	2 Mb/s	1 Mb/s	0.5W	$0.5 \mu J/bit$

$$e.j: \quad 10^{-6} \frac{s}{bit} \cdot 1 W = 10^{-6} \frac{W \cdot s}{bit}$$

$$coding \ gain = \frac{E_b \ (w/o \ coding)}{E_b \ (w/coding)} = 2 = 3dB$$

Exercise 10: Assuming one bit at a time is input into the encoder in the diagram above, what are k, n, K and the code rate?
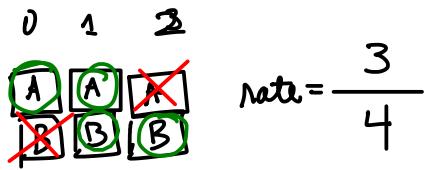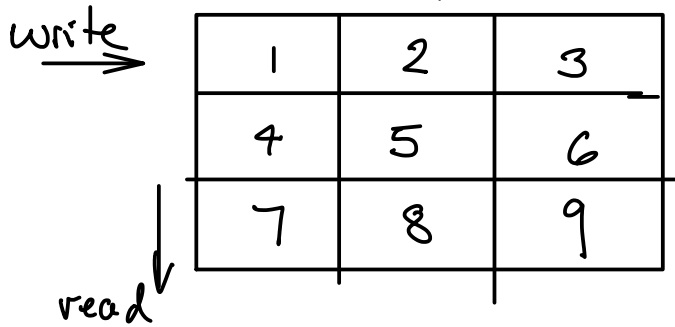
$$k = 1$$
$$n = 2$$
$$K = 7$$
$$\frac{k}{n} = \frac{1}{2}$$

Exercise 11: Consider the encoder above. If the only the bits corresponding to the outputs A, A and B, and B are transmitted corresponding to every three input bits, what is the code rate of this punctured code?
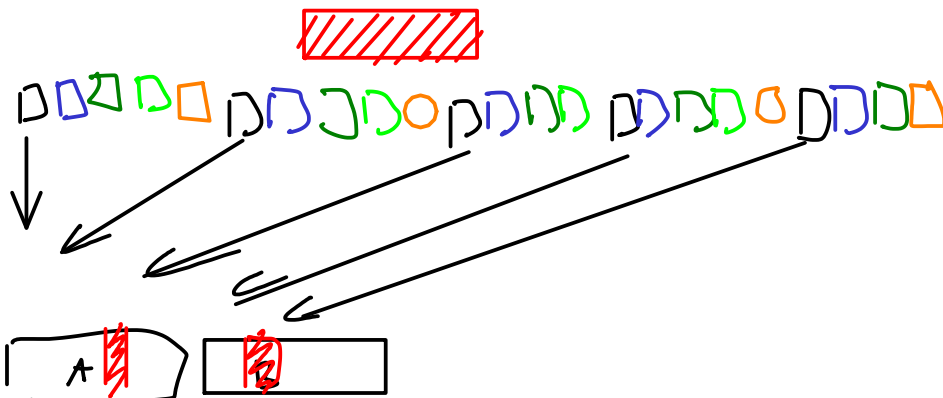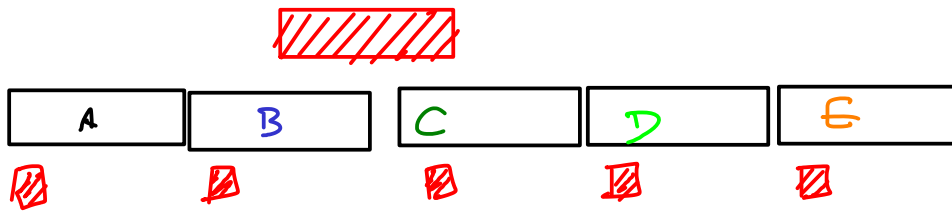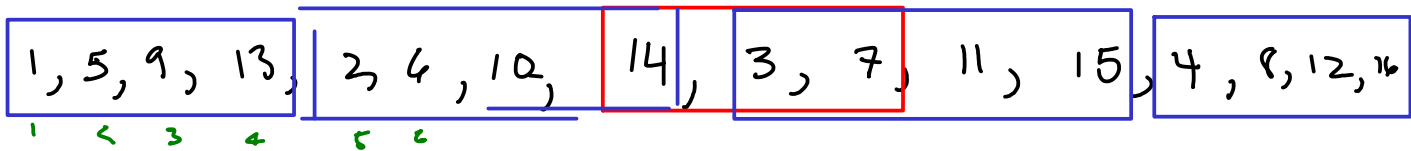
$$rate = \frac{3}{4}$$

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input: 1, 2, 3, ... 9

write →

interleaver

1	2	3
4	5	6
7	8	9

read ↓

output: 1, 4, 7, 2, 5, 8, 3, 6, 9

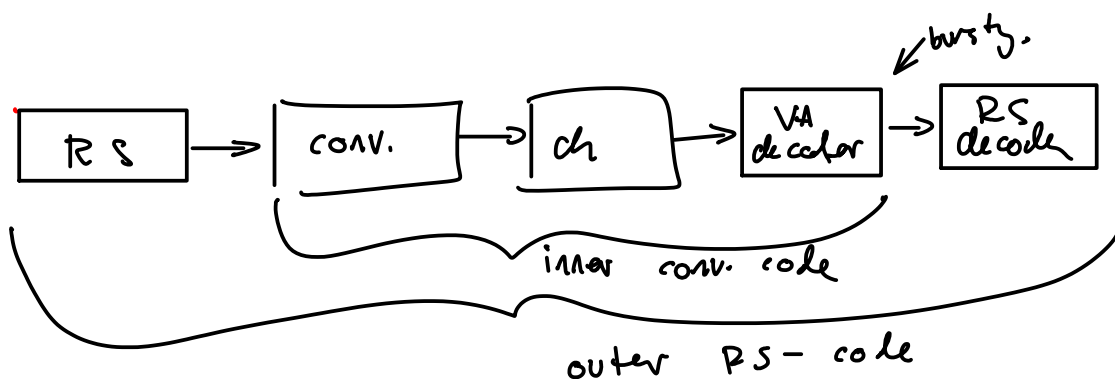| A	B	C	D	E

A	B

Exercise 12: Give the numbering of the bits coming out of a 4x4 interleaver. If bits 8, 9 and 10 of the interleaved sequence have errors, where would the errors appear in the de-interleaved sequence? If the receiver could correct up to one error per 4-bit word, would it be able to correct all the errors without interleaving? With interleaving?

1	2	③	4
5	6	⑦	8
9	10	11	12
13	⑭	15	16

$$1, 5, 9, 13, \quad 2, 6, 10, \quad 14, \quad 3, 7, 11, 15, \quad 4, 8, 12, 16$$

1 2 3 4   5 6

CONCATENATED CODES

bursty.

RS → CONV. → ch → VA decoder → RS decoder

inner conv. code

outer RS- code

Exercise 13: If errors on the channel happened in bursts and you were using a RS code using 8-bit words, would you want to interleave bits or bytes?

interleave bytes : because all the errors in one byte would be corrected, we want the errors concentrated into as few bytes as possible