Solutions to Assignment 3

Question 1

From the two circuits:

$$\frac{i_2}{i_1} = \frac{V/(R_{limit} + 20)}{V/(R_{limit} + 10)} = 0.8$$

so that:

$$\frac{R_{limit} + 10}{R_{limit} + 20} = 0.8$$

and we can solve for $R_{limit} = 30\ \Omega$.

The current-limiting resistor, the loop resistance and the load resistance form a voltage divider. For the maximum loop resistance of 10 ohms, the divider ratio is $\frac{10}{30+10+10} = 0.2$. Thus to ensure a swing of 1 V accross the load we would need a voltage swing at the source of $\frac{1}{0.2} = 5$ V.

If we had assumed zero loop resistance the divider ratio would have been $\frac{10}{30+10} = 0.25$ and the required source voltage swing would have been $\frac{1}{0.25} = 4$ V.

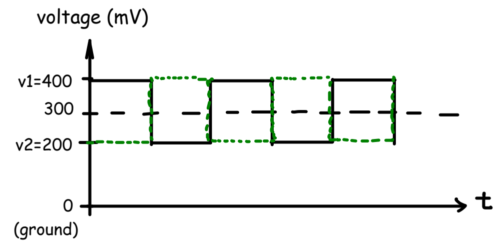
Question 2

The question is ambiguous because the common-mode and differential voltage only define two voltages with respect to ground, not the 4 voltages that would have to be defined for the two levels for each of the two input square waves.

However, a common case for a communication system is that the differential signal is bipolar (the differential voltages are ± 100 mV) while the common-mode voltage is fixed (always 300 mV).

In this case the common-mode voltage (300 mV) is the average of the two levels ($(v_1 + v_2)/2 = 300$) while the differential voltage (200 mV) is the difference between them ($v_1 - v_2 = 200$). So we can subsitute $v_2 = 200 + v_1$ into the first equation and solve for $v_1 = 400$ and $v_2 = 200$ mV.

The waveform would look as shown below.

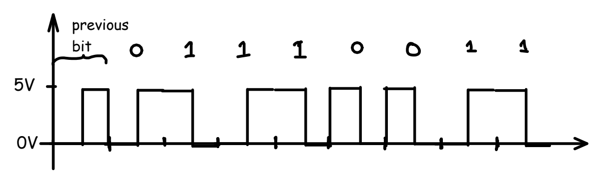


Question 3

The voltage ratio is the turns ratio so the turns ratio must be 4:1 (primary:secondary). The impedance ratio is the square of the turns ratio, 16:1, so the impedance seen at the primary would be 160 ohms.

Question 4

The waveform used to transmit the byte value 0x73 in MSB-first order using differential Manchester line coding and voltage levels of 0 and 5V is shown below. It is assumed that the previous bit was transmitted using a low-to-high transition. If the previous bit was transmitted as high-to-low the waveform would be inverted. The 0's are encoded as no change and the 1's are encoded as a change from the previous symbol.



Question 5

If a sequence of bits had bit-stuffing applied to prevent runs of 4 or more consecutive ones then a zero must be "stuffed" after every run of three consecutive ones. In the following sequence there is one run of three 1's:

```
110011101010
```

The original sequence without bit-stuffing must thus be:

```
11001111010
```

Question 6

The start of the frame begins with 0xA, 0x1. This is followed by the nybbles in the message with the 0xA nybble escaped by sending it twice: 0x1, 0x2, 0xA, 0xA, 0x4. The end of the frame is marked by sending 0xA, 0x2. Thus the complete transmitted frame is:

```
0xA, 0x1, 0x1, 0x2, 0xA, 0xA, 0x4, 0xA, 0x2
```

Question 7

64-QAM modulation transmits $\log_2(64) = 6$ bits per symbol. Transmission of a 1000-bit frame requires $1000/6 = 166\frac{2}{3}$ symbols. But the number of symbols transmitted must be a whole number, or $\lceil \frac{1000}{6} \rceil = 167$. To "fill up" these symbols $167 \times 6 = 1002$ bits are required so there will be $1002 - 1000 = 2$ padding bits.

Question 8

Each codeword has $n = 6$ bits. Since there are 4 codewords we are transmitting $k = \log_2(4) = 2$ data bits per codeword. The number of parity bits per codeword is $n - k = 6 - 2 = 4$. The code rate is $k/n = 2/6 = 1/3$.

To find the minimum distance of this code we need to find the minimum Hamming distance for all pairs of codewords. Calculating all of the distances (1:2, 1:3, 1:4, 2:3, 2:4 and 3:4) we get: 6, 3, 3, 3, 3, 6. The minimum is 3. A code with minimum distance $d = 3$ can detect $d - 1 = 2$ errors and correct $\lfloor \frac{d-1}{2} \rfloor = \lfloor \frac{3-1}{2} \rfloor = 1$ errors.

If the codeword 000011 was received, then the Hamming distances to each of the valid codewords are 5, 1, 4, and 2. Thus the codeword that was most likely to have been transmitted is the second (000111). Comparing the most likely transmitted (000111) and the received (000011) codewords the fourth bit would have been changed from 1 to 0.

Question 9

A coding gain of 2 dB means that the E_b/N_0 required to achieve a specified BER (10^{-9} in this case) is reduced by 2 dB when coding is used.

However, the use of rate-1/2 coding reduces the information rate by 1/2 and thus increases E_b by 3 dB (twice as much power is required per information bit because only half of the bits are data and the other half are parity).

Thus the required SNR must have decreased by $2 + 3 = 5$ dB by using coding and the required SNR is $13 - 5 = 8$ dB.

Question 10

If the bit sequence 100100 1110 is punctured by dropping the second and third of every six consecutive encoded bits the result is 1100 10.

6 data bits would be generated by a rate-1/2 code applied to 3 data bits. If we transmit 4 coded bits for every 3 information bits the code rate is 3/4 (information bits/transmitted bits).

Question 11

The generator polynomial $G(x) = x^2 + 1$ as a bit sequence is 101. The remainder will thus have two bits and we need to extend the message by two bits to include the remainder (which is the CRC).

The division of the message by $G(x)$ is:

```
111100
101
---
 101
 101
 ---
  000
  000
  ---
   000
   000
   ---
    00
```

and the remainder (CRC) is 00. In this case the CRC is the same as the zeros used to extend the CRC so the check of the CRC is exactly the same as computing the CRC initially and gives a result of 0 indicating no errors.