Solutions to Assignment 2

Question 1

The wire diameter doubles for every decrease of 6 in the wire gauge. 24-gauge wire is about 0.5 mm diameter so the diameter of 22 AWG wire will be $2^{\frac{2}{6}} = 1.26$ times larger or ≈ 0.63 mm.

The cross-sectional area is $\pi r^2 = 3.14 \times 0.32^2 \approx 0.31\text{mm}^2$.

The total cross-sectional area of stranded 7x22 wire is 7 times this or about 2.2 mm^2.

A solid wire diameter of diameter d has an area $A = \pi(d/2)^2$ so we can solve for $d = 2\sqrt{A/\pi} = 2 \times \sqrt{2.2/\pi} = 1.7$ mm. The decrease in wire gauge is $6\log_2(1.7/0.63) = 8.5$ so 7x22 wire is equivalent to about $22 - 8 = 14$ AWG.

Question 2

The characteristic impedance of co-ax is given by:

$$Z_0 \approx \frac{60}{\sqrt{\varepsilon_r}} \ln\left(\frac{D}{d}\right)$$

Since the characteristic impedance, $Z_0 = 100$, the inner conductor diameter, $d = 0.5$ mm, and the outer conductor diameter $D = 2.5$ mm, are given we can solve for ε_r:

$$\varepsilon_r \approx \left[\frac{60}{Z_0} \ln\left(\frac{D}{d}\right)\right]^2 \approx 0.93$$

and the velocity of propagation is related to the dielectric constant by:

$$c \times VF = \frac{c}{\sqrt{\varepsilon_r}} = \frac{3 \times 10^8}{\sqrt{0.93}} \approx 3.1 \text{ m/s}$$

which is clearly impossible (faster than light!) so the cable specified is not realizable.

Question 3

If the cut-off frequency, f_c, is inversely proportional to the outer diameter of the co-ax and for 1.2 mm co-ax $f_c = 110$ GHz then:

$$\frac{f_c}{110} = \frac{1.2}{5}$$

and we can solve for $f_c = 26.4$ GHz.

Question 4

A run of 480 m in lengths of 200 m will require 3 lengths and two splices which will add an additional 2 m of cable (total of 482 m) and $2 \times 0.7 = 1.4$ dB of loss.

Since the total loss at 150 MHz must be less than 25 dB, the loss of the cable itself must be less than $25 - 1.4 = 23.6$ dB. The cable must therefore have loss of less than $23.6/4.82 = 4.92$ dB/100m.

Of the LMR cables listed in the specified data sheet the LMR-400 cable would meet the requirements at the lowest cost ($2.10/m).

If we round up to the nearest multiple of 50 m, we would have to buy 500 m of cable which would cost $1050.

Question 5

If the path loss can be approximated by the formula:

$$\frac{P_R}{P_T} = L_0 \left(\frac{d}{d_0}\right)^{-n}$$

then for $L_0 = -60$ dB (it is a loss, not a gain), $d_0 = 50$ m and $n = 3.3$, the path loss at $d = 1$ km would be

$$10^{\frac{-60}{10}} \left(\frac{1000}{50}\right)^{-3.3} = -103 \text{ dB}$$

Many people assumed the path gain was +60 dB instead of -60 dB because of the way the question was worded. This would have resulted in a path loss 120 dB lower or a gain of 17dB (not possible).

Question 6

Metallic transmission lines of the types we have studied (co-ax and twisted pair) pass DC signals but have a loss that increases with frequency. Thus they could be considered low-pass channels.

However, optical fiber and waveguides don't pass frequencies below a certain frequency and could be considered band-pass channels.

Question 7

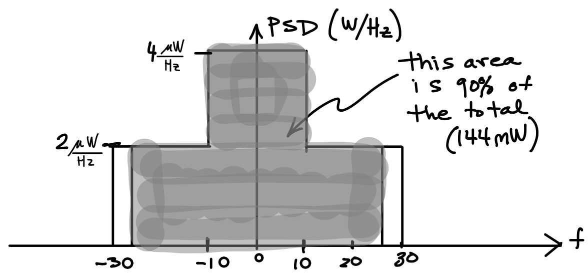
The total power is the area under the power spectral density curve (PSD times the bandwidth). The area can be computed in three sections, each of which has a bandwidth of 20 kHz (20×10^3 Hz) and a PSD of 2×10^{-6} or 4×10^{-6} W/Hz:

$$(2 + 4 + 2) \times 10^{-6} \times 20 \times 10^3 = 0.16 \text{ W}$$

or 160 mW.

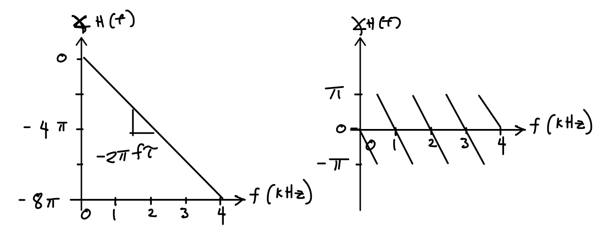
To compute the 90% power bandwidth we need to find the bandwidth containing containing 90% of this power or 144 mW. If this bandwidth is centered on the PSD plot, then we need to find the frequency range (bandwidth) that multiplied by the 2μW/Hz PSD contains 16 mW of power (10%).

If this bandwidth is B, $B \times 2 \times 10^{-6} = 16 \times 10^{-3}$ and $B = 8$ kHz. Thus the 90% power bandwidth is $60 - 8 = 52$ kHz (from -26 kHz to +26 kHz):



Question 8

If the channel's only effect is to delay the signal, the only effect will the to change the phase of each frequency component. A frequency component at frequency f will shift in phase by $-2\pi f\tau$ due to a delay of τ. When the phase shift is plotted versus frequency this is a straight line with a slope $-2\pi\tau$. In this case $\tau = 1 \times 10^{-3}$ and f ranges from 0 to 4×10^3 so the phase shift will range from 0 to -8π. The phase can also be drawn "wrapped" to a range of $-\pi$ to π:



Question 9

A full-wave rectifier causes the polarity of the signal voltage to be changed to be all positive. Like clipping, this is an irreversible change to the signal and cannot be undone by changing the amplitude and phase of the frequency components. Therefore the distortion caused by a rectifier is a type of non-linear distortion.

Question 10

The third-order distortion products at *positive* frequencies are at $2f_1 - f_2$ and $2f_2 - f_1$ which are $2 \times 1 - 1.2 = 0.8$ kHz and $2 \times 1.2 - 1 = 1.4$ kHz.

If the input levels are 0 dBm, the amplifier gain is 20 dB and the third-order products are 30 dB down (less than) the output levels of the desired components, then the 3rd-order product levels will be $0 + 20 - 30 = -10$ dBm.

Note that the third harmonics (3 kHz and 3.6 kHz) are not IMD products.

Question 11

The THD is the ratio of the harmonic power to the desired signal power. In this case the input signal is a sinusoid which has only one frequency component. The notch filter removes the desired signal leaving only the harmonic distortion components. If the signal power is 1 W and the harmonic distortion power is 10 mW, the the THD is $10 \log(0.01/1) = -20$ dB.

Question 12

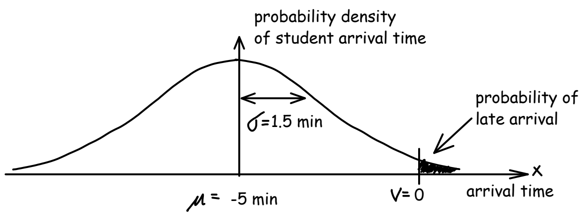
The noise output power for an gain amplifier with a gain G is given by $kTBFG$. In at standard temperature (290 K) $kT = -174$ dBm/Hz and the noise power can be computed as $-174+10\log(B)+F(dB)+G(dB) = -174+10\log(10^7)+2+8 = -94$ dBm.

If the amplifier input level is -100 dBm and the gain is 8 dB the output level will be -92 dBm.

The SNR (in dB) is the difference, or 2 dB.

Question 13

If we consider a student's arrival time relative to the class start time as a Gaussian variable, we are looking for the probability that the arrival time is greater than 0 ($v = 0$):



We are told the arrival time distribution has a mean $\mu = -5$ (on average they arrive 5 minutes before the start of the class) and a standard deviation $\sigma = 1.5$ minutes. The normalized Gaussian threshold variable t is thus $t = \frac{v-\mu}{\sigma} = \frac{0-(-5)}{1.5} \approx 3.3$.

We can find the approximate value of $F(t)$, the probability that the arrival time is *less than* t using the graph on page 6 of the notes for Lecture 3 (about 99.95%), the Logistic function approximation (99.65%) or a calculator (99.96%). This is the probability that students arrive before the start of the class ("on time"). The probability that students arrive late is the complement of this or about 0.05% (only 1 in 2000 arrivals!).

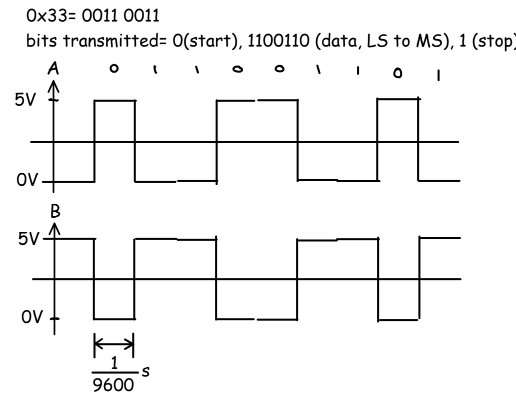
Question 14

The RS-422 waveforms used to transmit the byte value 0x33 has the same format as the RS-232 signal: a start bit, 7 or 8 data bits in order from LS to MS bit and a stop bit.

The polarities are defined in the V.11 (RS-422) standard on the course web site (Section 4) as:

polarity	data	level
$A > B$	0	space
$A < B$	1	mark

If the differential voltage 5 V and the common-mode voltage is 2.5 V then relative to ground the two voltages are 0 and 5V. The bit duration is the inverse of the bit rate, $\frac{1}{9600}$ s.

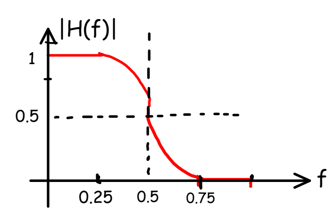


If the interface voltages appear on pins labelled TxD+ and TxD- (transmit data) then these pins are outputs. Since TxD is an output on a DTE this interface is wired as a DTE. The interface's RxD pins would be inputs.

Question 15

If the channel can be used to signal at a maximum symbol rate of 1 MHz without ISI, then *half* of this (0.5 MHz)must be the -6 dB bandwidth.

If the excess bandwidth parameter, $\alpha = 0.5$ is the fraction of this minimum bandwidth that is in excess of the minimum. This is 0.5×0.5 MHz $= 0.25$ MHz. The channel response will thus extend to $0.5+0.25 = 0.75$ MHz. The response must be symmetrical about the -6 dB frequency so the response will also taper off up to $0.5-0.25 = 0.25$ MHz. A sketch of one possible transfer function is:



The frequency at which the magnitude of the transfer function have a value 0.71 (-3 dB) of the maximum will depend on the specific frequency response but it must be between 0.25 and 0.5 MHz. A reasonable value would be about 0.375 MHz.

Question 16

If the sampling rate is $f_s = 256$ kHz and there are $N = 256$ samples per OFDM block then the symbol duration must be $T_s = \frac{256}{256 \times 10^3} = 1$ ms. The subcarrier spacing is the inverse of the symbol duration or 1 kHz.