Lecture 9

Exercise 1: Compute the modulo-4 (3-bit) checksum of a frame with values 4, 1, and 3. Would an error be detected if the received frame was 4, 1, 3, 0? How about if the received frame was 1, 4, 3?

$4+1+3=8$

$\frac{8}{4} = 2$ remainder 0

checksum $= -0 = 0$ so we transmit : $\boxed{4, 1, 3, 0}$

$4+1+3+0+\underbrace{(-0)}_{\text{checksum}} = 8$ 8 modulo 4 = 0 ⇒ no error detected

$1+4+3+(-0) = 8$ 8 modulo 4 = 0 ⇒ no error detected!

$5+1+3+(-0) = 9$ 9 modulo 4 = 1 ⇒ there was an error

Exercise 2: What is a modulo-2 sum? What is the modulo-2 sum of 1, 0 and 1? What is the modulo-2 sum if the number of 1's is an even number?

The modulo-2 sum is remainder after dividing the sum by 2.

$1+0+1 = 2$ $\frac{2}{2} = 0$ remainder 0 ← modulo-2 sum.

if # of 1's is even the modulo-2 sum is zero (0).

$1+2+3 = 6$ 6 modulo 4 = 2 $\frac{6}{4} = 1$ remainder 2

need to append -2 as the checksum

so we send

$[1, 2, 3, -2]$

$1+2+3-2 = 4$ 4 modulo 4 = 0

∴ we conclude no errors.

Exercise 3: How many different code words does an (n, k) code have?

there are k data bits so there are 2^k possible frames (codewords).

Exercise 4: How many different patterns of $n - k$ parity bits are there? Assuming all parity bit sequences are equally likely, what is the probability that a randomly-chosen code word has the same parity bits as another codeword?

$0\ 0\ 0\ 0\ 1\ \overbrace{\qquad\qquad}^{n-k}$

$$\text{prob of getting same parity bits} = \frac{1}{2^{n-k}}$$

In general 2^{n-k} different sets of n-k bits

but in the code we have 2^k codewords.

e.g. $k=3$ $\quad n-k=3$

$n-k=3$

$n=6$

$$2^3 = 8 \left\{ \begin{array}{l|l} 000 & 111 \\ 001 & 110 \\ 010 & 101 \\ \vdots & \\ 111 & 000 \end{array} \right.$$

odd parity

$n-k=1$

$$\begin{array}{l|l} 000 & 1 \\ 001 & 0 \\ 010 & 0 \\ 011 & 1 \\ \vdots & \vdots \\ 111 & 0 \end{array}$$

$2^k = 8$

$2^{n-1} = 2$

→ different codewords could have same parity

Exercise 5: What is the Hamming distance between the codewords 11100 and 11011?

$$\begin{array}{r} 1\ 1\ 1\ 0\ 0 \\ \oplus\ 1\ 1\ 0\ 1\ 1 \\ \hline 0+0+1+1+1 = 3 \end{array}$$

Exercise 6: A block code has two valid codewords, 101 and 010. The receiver receives the codeword 110. What is the Hamming distance between the received codeword and each of the valid codewords? What codeword should the receiver decide was sent?

possible codewords $\left\{ \begin{array}{l} 101 \\ 010 \end{array} \right.$

110 ← received

$$\begin{array}{r} 1\ 0\ 1 \\ \oplus\ 1\ 1\ 0 \\ \hline 0+1+1 = 2 \end{array}$$

$$\begin{array}{r} 0\ 1\ 0 \\ \oplus\ 1\ 1\ 0 \\ \hline 1+0+0 = 1 \end{array}$$

← probably this was transmitted (minimum Hamming distance)

Exercise 7: Assume errors on a channel cause a frame error rate of 50%. When a rate-1/2 FEC code is used the frame error rate drops to 1%. Compute the throughputs with and without coding relative to the uncoded and error-free channel. What other advantages might the use of FEC provide?

	uncoded	coded	ideal uncoded, error-free
code rate	1 $(k=n)$	$\frac{1}{2}$	1
frame error rate	$\frac{1}{2}$	1%	0
throughput (data bits/second)	$1 \times (1-\frac{1}{2}) = \frac{1}{2}$	$\frac{1}{2} \times (1-0.01) = \frac{1}{2} \times 0.99 = 0.495$	$1 \cdot (1-0) = 1$

$$\text{throughput} = \text{code rate} \times \underbrace{\left(1 - \text{frame error rate}\right)}_{\text{fraction of frames correctly received}}$$

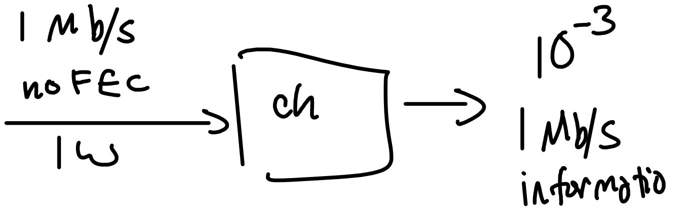
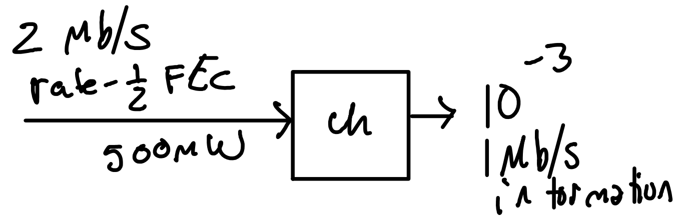
(assuming no retransmissions)

$$\underbrace{\underbrace{1 + \frac{1}{2} + \frac{1}{4} + \cdots}_{\gg \frac{1}{2}} \quad \Bigg| \quad \underbrace{1 + 0.01 + .0001 + \cdots}_{\approx 0.5} \quad \Bigg| \quad 1}_{\text{total number of transmisson } \underline{\text{including}} \text{ retransmissions}}$$

Exercise 8: A system without coding needs to transmit at 1W to transmit 1 Mb/s at an error rate of 10^{-3}. When a rate-1/2 code is used the power to transmit the necessary 2Mb/s of data and parity bits decreases to 500mW. What is the channel bit rate in each case? What is the information rate in each case? What is E_b? What is the coding gain?

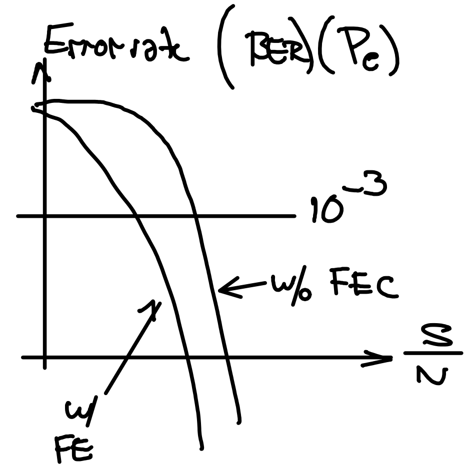
Energy = Power x Time

Joules = Watts x Seconds

	Channel bit rate	information rate	E_b
	1 Mb/s	1 Mb/s	$1W \cdot \frac{1}{10^6}$ $= 1\mu J/bit$
	2 Mb/s	1 Mb/s $R = \frac{k}{n} = \frac{1}{2}$ $= R \cdot 2 Mb/s$ $= 1 Mb/s$	$\frac{1}{2}W \cdot \frac{1}{10^6}$ $= 0.5\mu J/bit$

$$\text{coding gain} = \frac{E_b \text{ w/o coding}}{E_b \text{ w/ coding}}$$

$$= \frac{1}{0.5} = 3dB.$$



Exercise 9: Assuming one bit at a time is input into the encoder in the diagram above, what are k, n, K and the code rate?

$k = 1$ input at a time

$n = 2$ outputs "

$K = 6 + 1 = 7$ bits used in calculating the output

Exercise 10: Consider the encoder above. If the only the bits corresponding to the outputs A, A and B, and B are transmitted corresponding to every three input bits, what is the code rate of this punctured code?

A	A 1	A 3	A 5
B	B 2	B 4	B 6

for rate ½ transmit

A B A B A B
1 2 3 4 5 6

for rate $\frac{3}{4}$ transmit:

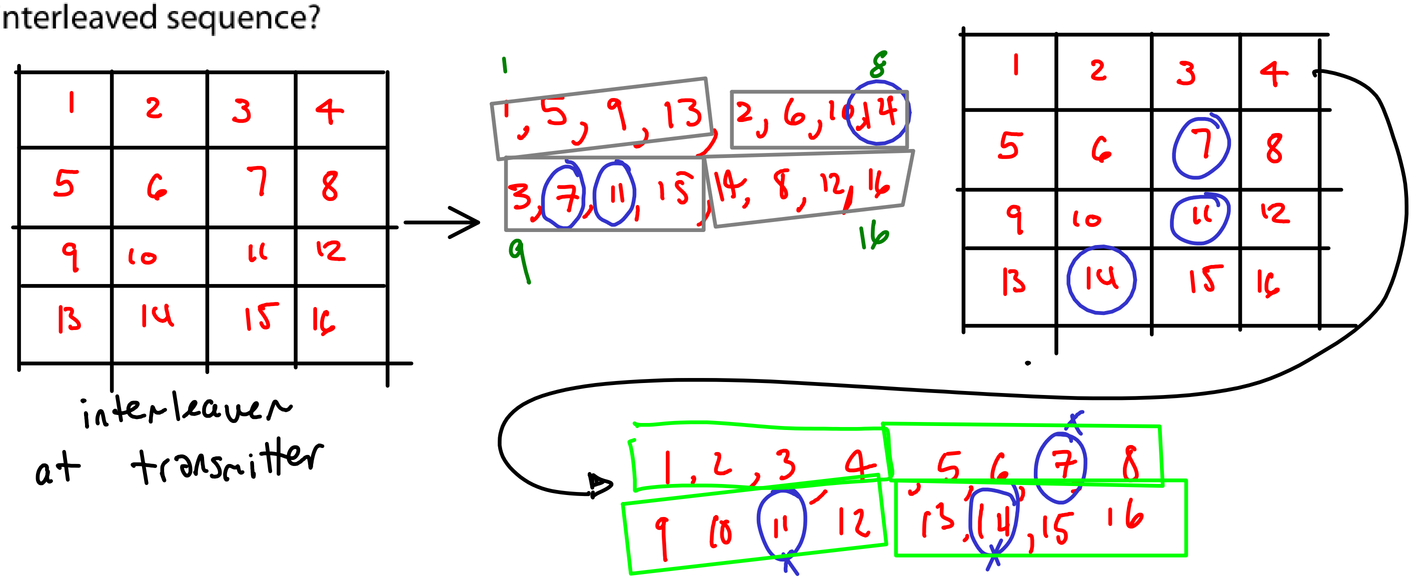
A, A B, B
1 2 3 4

$$\begin{array}{r} 1\ 0\ 1 \\ \oplus\ 1\ ?\ 1 \\ \hline 0+?+0 = 0 \end{array} \qquad \begin{array}{r} 1\ 0\ 1 \\ 1\ ?\ 0 \\ \hline 0+?+1 = 1 \end{array}$$

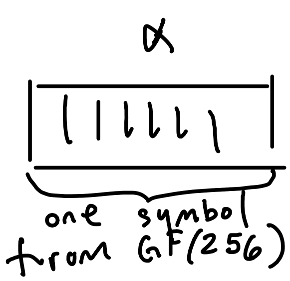
$$\frac{k}{n} = \frac{3 \text{ input bits}}{4 \text{ output bits}}$$

when computing distances, ignore erasures (?) ← don't know, value not transmitted

Exercise 11: Give the numbering of the bits coming out of a 4x4 interleaver. If bits 8, 10 and 11 of the interleaved sequence have errors, where would the errors appear in the de-interleaved sequence?



Exercise 12: If errors on the channel happened in bursts and you were using a RS code using 8-bit words, would you want to interleave bits or bytes?



interleave bytes because bits in each byte are all "corrected" together.

GF(256) is a "byte-correcting" code