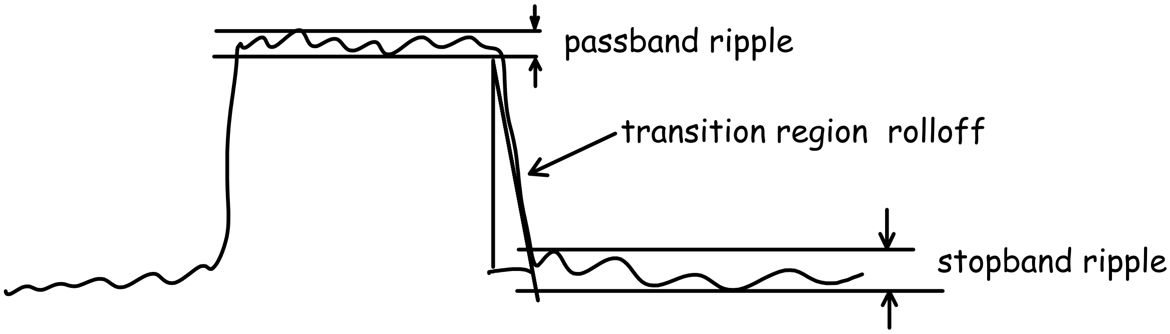
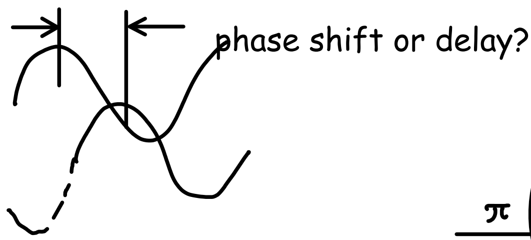
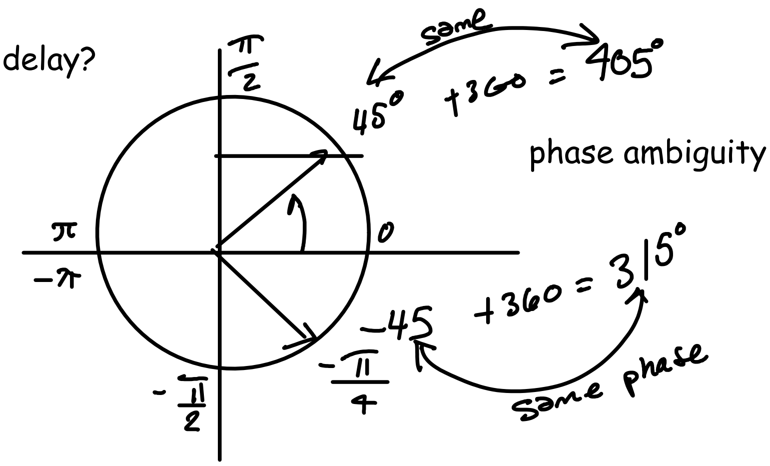
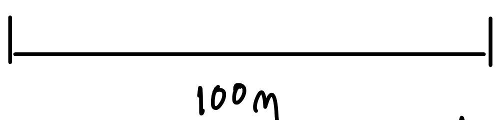
Lecture 3





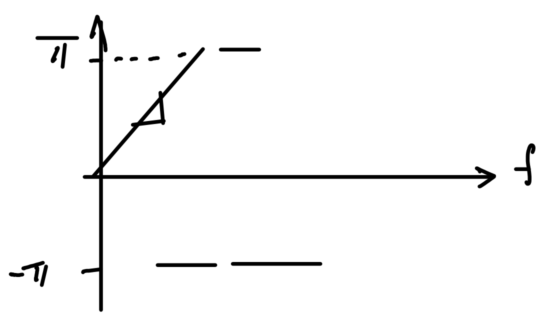


Exercise 1: A 100m transmission line has a velocity factor of 0.66. Plot the phase response of the cable over the frequency range 0 to 6 MHz.



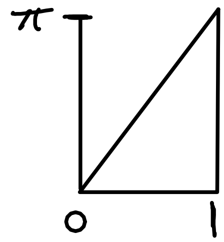
$VF = 0.66$

$v = VF \cdot c$
$= 0.66 \cdot 3\times10^8$
$= 2\times10^8$



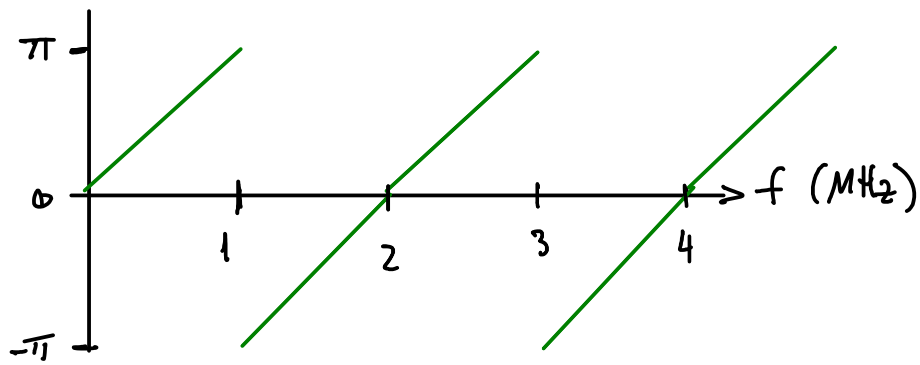
$$\tau = \frac{d}{v} = \frac{100}{2\times10^8} = 50\times10^{-8} = 500\,\text{ns}$$

$$\theta = 2\pi f \tau$$
$$= 2\pi f \cdot 500\times10^{-9}$$



slope $= 2\pi\tau$
$= 1\times10^{-6}\pi$

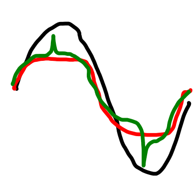
$$\cancel{\pi} = 2\cancel{\pi} f \cdot 500\times10^{-9}$$
$$f \quad \frac{1}{2\cdot 500\times10^{-9}} \simeq 10^{6} = 1\,MHz$$



Exercise 2: A telephone line is being used to transmit symbols at a rate of 300 symbols/second. If the group delay must be less than 10% of the symbol period, what is the maximum allowable group delay?

$$f_{symbol} = 300$$
$$T_{symbol} = \frac{1}{300} = 3.3\,ms$$

maximum group delay $= 10\%$ of $T_{symbol} = 0.33\,ms$
$\approx \underline{330\,\mu s}$



Exercise 3: The input to a non-ideal amplifier is the sum of two sine waves at frequencies of 1 and 1.2 MHz. What are the frequencies of the even harmonics of these frequencies? Of the odd harmonics? What are the frequencies of the third-order IMD products?

$f_1 = 1\,MHz$

$f_2 = 1.2\,MHz$

even harmonics: $2f_1, 4f_1, \dots = 2, 4, 6, \dots$ MHz

$2f_2, 4f_2, \dots = 2.2, 4.8, 7.2, \dots$ MHz

odd harmonics: $3, 5, 7, \dots$

$3.6, 6, 8.4, \dots$

3rd-order IMD product frequencies:

n	m	$nf_1 \pm mf_2$	
1	2	$1+2.2$	3.2
		$1-2.2$	-1.2
2	1	$2+1.2$	3.2
		$2-1.2$	0.8

$\pm 1 \quad \pm 2$

$+1 \quad +2$

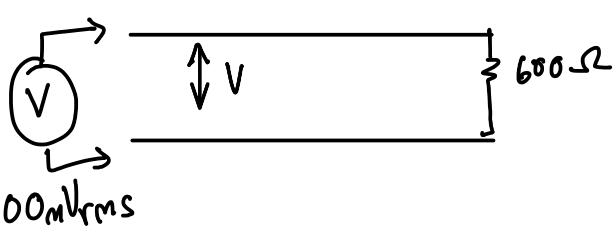
$+1 \quad -2$

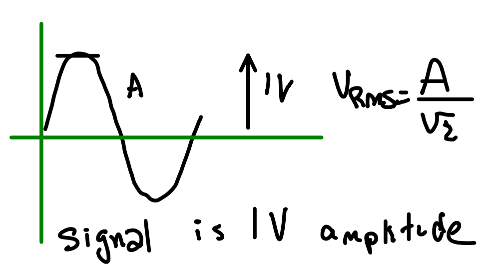
$-1 \quad +2$

$-1 \quad -2$

n m

Exercise 4: A sinusoidal signal is being transmitted over a noisy telephone channel. The voltage of the signal is measured with an oscilloscope and is found to have a peak voltage of 1V. Nearby machinery in inducing a noise voltage onto the line. The voltage of this noise signal is measured with an RMS voltmeter as 100mVrms. The characteristic impedance of the line is 600Ω and it is terminated with that impedance. Why was an RMS voltmeter used? What is the signal power? What is the noise power? What is the SNR?





- used RMS voltmeter because normal voltmeter only reads RMS correctly for sine waves.

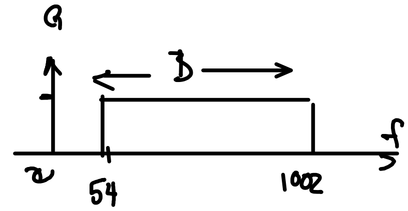
- $S = \frac{V_{rms}^2}{R} = \frac{(1 \times 0.707)^2}{600} = \frac{1}{1200} \approx 1\,mW$

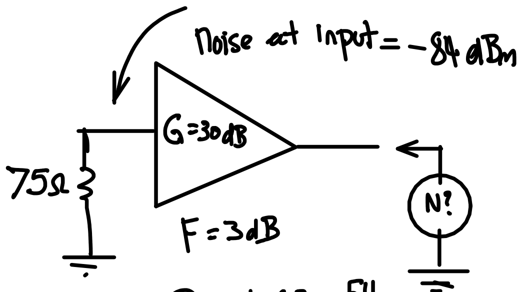
- $N = \frac{V_{rms}^2}{R} = \frac{(0.1)^2}{600} = \frac{10^{-2}}{600} \approx \frac{10^{-2}}{10^3} \approx 10^{-5}\,W$ $\quad 1.6 \times 10^{-5}$

$$\frac{S}{N} \approx \frac{1\times10^{-3}}{1\times10^{-5}} \approx 10^{2} \qquad \left(\frac{\frac{1}{1200}}{\frac{10^{-2}}{600}} = \right)$$

$$\frac{S}{N}\ (\text{in dB}) = 10\log\left(\frac{S}{N}\right) = 10\log(100) = 20$$

Exercise 5: A line amplifier for a cable TV system amplifies the range of frequencies from 54-1002 MHz. The amplifier has a gain of 30 dB and a noise figure of 3 dB. If we connect a 75Ω resistor (the input impedance of the amplifier) to the input how much power will we measure at the output of the amplifier?





$$\underset{\text{dBm}/\text{Hz}}{N_{dBm} = -174} + \underset{\text{dB}\cdot\text{Hz}}{10\log(B)} + \underbrace{10\log(F)}_{\text{dB, due to amplifier}}$$

$$= -174 + 90 +$$

Noise at input = −84 dBm

$B = 1002 - 54$
$\approx 1000\ \text{MHz}$
$= 10^{9}\ \text{Hz}$

$10\log B = 10\log(10^{9}) = 10\times 9 = 90$

Noise at output =

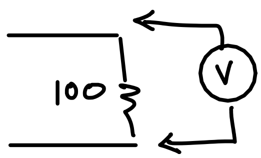
$$-174 + 10\log(B) + 10\log(F) + G \ (\text{dB})$$

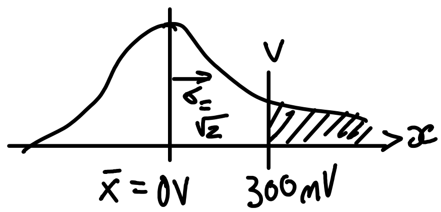
$$= -84 + 3 + 30$$

$$= -51\ \text{dBm}$$

Exercise 6: The output of a noise source has a Gaussian (normally) distributed output voltage. The (rms) output power is 20mW and the output impedance is 100Ω. What fraction of the time does the output voltage exceed 300mV? Hint: the variance (σ^2) of a signal is the same as the square of its RMS voltage.

not given: average value (DC voltage)
$\Rightarrow$ assume 0V.





$$t = \frac{v - \bar{x}}{\sigma} = \frac{0.3-0}{\sqrt{2}} = \frac{0.3}{\sqrt{2}}$$
$$\approx 0.2$$

$$20mW = \frac{V_{rms}^2}{R}$$

$$R = 100\,\Omega$$

$\sigma^2 = ?$

$\sigma = ?$

$$\sigma^2 = V_{rms}^2 \qquad \sigma = V_{rms}$$

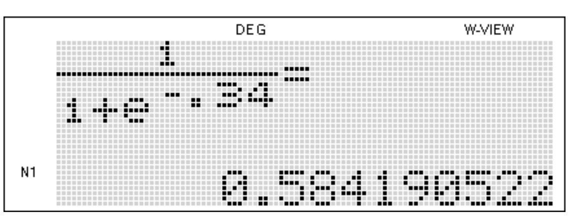
$$20mW = \frac{V_{rms}^2}{100}$$

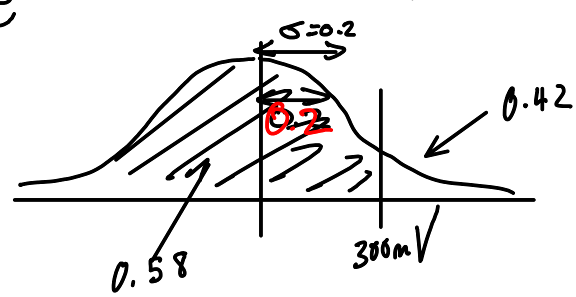
$$V_{rms}^2 = 2$$

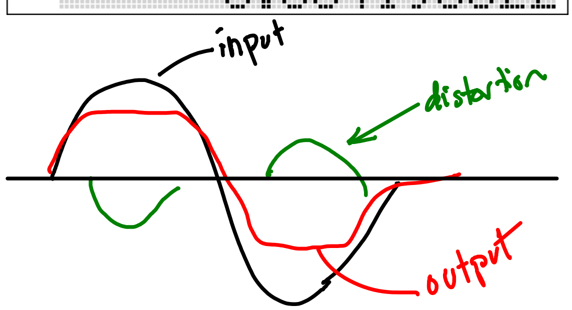
$$V_{rms} = \sqrt{2} = \sigma$$

Using the Logistic fcn. approximation: $F(t) = \dfrac{1}{1+e^{-1.7t}}$

$$P(x < 300mV) \simeq \frac{1}{1+e^{-1.7\times 0.2}} \simeq \frac{1}{1+e^{-0.34}} = 0.58$$







$$SIR = \frac{S}{I}$$

$$SINR = \frac{S}{I+N}$$

$$SINAD = \frac{S}{I+N+D}$$