Inter-Symbol Interference

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What limits the speed at which we can transmit data over a channel?

Waveforms transmitted over a bandwidth-limited channel tend to get "smeared" in time. The channel extends the duration of each transmitted symbol in time and possibly into subsequently-transmitted symbols. This means there is a possibility that each symbol will interfere with subsequent symbols. This interference is called inter-symbol interference (ISI).

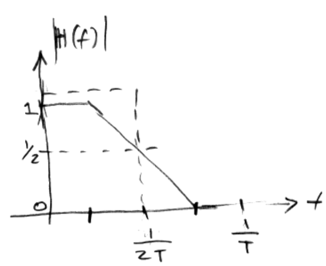
Nyquist no-ISI Criteria

It is possible to derive conditions for the channel's frequency-domain transfer function that results in no ISI at times that are multiples of the symbol period. One condition that results in no ISI is that the channel frequency response have odd symmetry around half of the symbol frequency. This condition is:

$$H(\frac{1}{2T}+f)+H(\frac{1}{2T}-f)=1 \text{ for } 0<f<\tfrac{1}{2T}$$

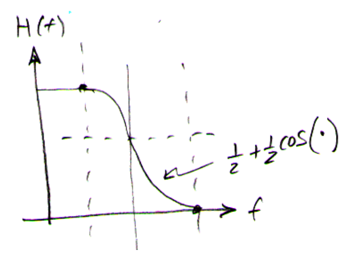
Exercise 1: What is the transfer function of a channel with infinite bandwidth? Does this channel meet the Nyquist no-ISI condition?

There are many no-ISI transfer functions. For example, the following two straight-line transfer functions meet the no-ISI condition:



The dashed line is a "brick-wall" filter whose response is 1 below half of the symbol rate and zero above that. Although such a filter would have the minimum overall bandwidth, it is not physically realizable and has other problems as described below.

The filter with the linear transfer function is also difficult to implement. A more practical transfer function is the so-called raised-cosine function which is a half-cycle of a cosine function centered around half of the symbol rate.



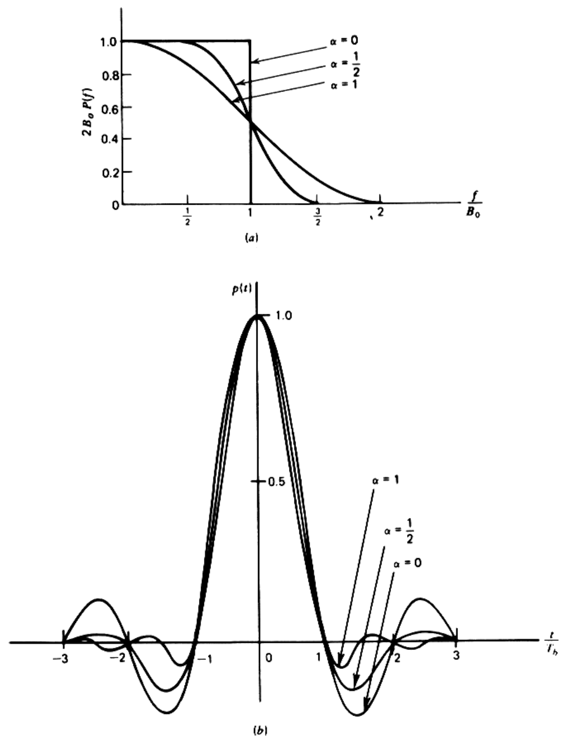
Note that it is symmetry about $1/2T$ that ensures there will be no ISI rather than the exact filter shape. Thus we are free to implement other transfer functions, possibly arbitrary ones, if they make the implementation easier.

Excess Bandwidth

However, the width of the transfer function in the transition between passband and stopband does have an impact on the shape of the impulse response and on the sensitivity of the receiver to errors in the timing of the sampling point.

This parameter, α, is called the "excess bandwidth". The following diagram[1] shows how the excess bandwidth parameter for a raised-cosine transfer function affects the impulse response.

[1] From Simon Haykin, "An Introduction to Analog and Digital Communication", 1989.



Larger values of excess bandwidth result in smaller values of the impulse response which in turn reduces the amount of ISI near the sampling point. This makes the receiver less sensitive to errors in the timing of the sampling point.

Exercise 2: What is the possible range of values of α?

Equalization

To avoid ISI, the total channel response including the transmit filters, the channel and the receiver filter(s) have to meet the Nyquist no-ISI condition.

When the channel by itself is unlikely to meet the no-ISI conditions, the transmitter and receiver typically use filters, known as "equalizers", to ensure the no-ISI condition is met.

Transmitter and receiver filters typically have other functions beside equalization. For example, the transmit filter may limit the bandwidth of the transmitted signal to limit interference to adjacent channels. The receiver filter may filter out noise and interference from adjacent channels and thus improve the SIR and SNR. The communication system designer would design the transmitter and receiver filters to meet both the filtering and equalization requirements.

A common situation is a flat channel where interference is not an issue. In this case a reasonable approach is to put half of the filtering at the transmitter and half at the receiver. In order to achieve an overall raised cosine transfer function, each side has to use a "root raised cosine" (RRC) transfer function. The product of the two filters is thus the desired raised-cosine function which meets the no-ISI condition.

Exercise 3: Could equalization be done at the transmitter only? Why or why not?

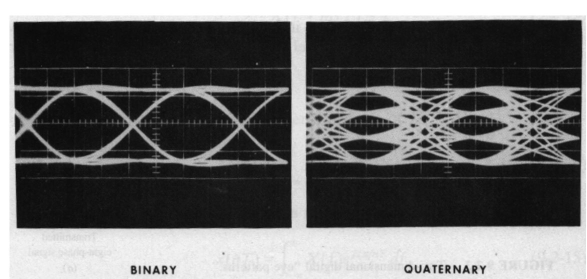
Adaptive Equalizers

In many communication systems the transfer function of the channel cannot be predicted ahead of time. One example is a modem used over the public switched telephone network (PSTN). Each phone call will result in a channel that includes different "loops" and thus different frequency responses. Another example is multipath propagation in wireless networks. The channel impulse response changes as the receiver, transmitter or objects in the environment move around.

To compensate for the time-varying channel impulse response the receiver can be designed to adjust the receiver equalizer filter response using various algorithms.

Eye Diagrams

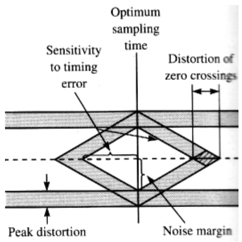
An eye diagram is superimposed plots of duration T of the received waveform (for random data). The eye diagram graphically demonstrates the effect of ISI. Some examples of eye diagrams produced by an appropriately-triggered oscilloscope[2]:



[2]From John G Proakis, "Digital Communications", 3rd Ed., 1983.

The vertical opening at the sampling time, called the "eye opening", represents the amount of ISI at the sampling point.

The horizontal opening indicates how sensitive the receiver would be to errors in sampling point timing[3]:



OFDM

And alternative to adaptive equalization is to use a technique called Orthogonal Frequency Division Multiplexing (OFDM). OFDM transmits many (typically 64 or more) symbols in parallel at different "sub-carrier" frequencies. This reduces the symbol period by an amount equal to the number of parallel streams without reducing the overall bit rate. OFDM systems insert a "guard time" between symbols that is longer than the duration of the impulse response of the channel. This minimizes interference between symbols. OFDM has become more popular than adaptive equalization because it is simpler to implement and more robust. OFDM is used by most ADSL, WLAN and 4G cellular standards.

ISI and Shanon Capacity Bounds

Note that the symbol rate limitations resulting from ISI do not by themselves limit the achievable bit rate or the capacity of the channel. For example, once the ISI is controlled we can use an arbitrarily large symbol set (any value of M) and transmit any number of bits per symbol.

The limitation on symbol rate is also different than the constraint on channel capacity defined by the Shanon bound which takes into account the signal to noise ratio as well as the bandwidth.

[3]Proakis, op. cit.