Capacity

The Shannon capacity of a channel is the information rate above which it is not possible to transmit data with an arbitrarily low error rate. For the AWGN channel the capacity is given by:

$$C = B*\log_2(1+ S/N)$$

$$\log_2(x) = \frac{\log_{10}(x)}{\log_{10}(2)}$$

The Shannon limit does not say that you can't transmit data faster than this limit, only that if you do, you can't reduce the error rate to an arbitrarily low value.

The Shannon limit does not specify how to achieve capacity, for example, what modulation and coding to use. However, Shannon work does hint that using error-correcting codes should allow us to achieve arbitrarily low error rates as long as we limit the data (actually, information) rate to less than the channel capacity.

Exercise: What is the capacity of a 3 kHz channel with an SNR of 20dB?

$$\frac{S}{N} = 20\,dB$$

$$= 10^{\left(\frac{20}{10}\right)} = 10^2$$

$$= 100$$

$$C = B \log_2\left(1+\frac{S}{N}\right)$$

$$B = 3000$$

$$\frac{S}{N} = 100$$

$$C = 3000 \log_2(1+100)$$

$$\approx \underbrace{\qquad\qquad}_{\approx 6.5}$$

$$\approx 20{,}000 \text{ bps}$$

$$(19{,}975)$$

To implement systems that operate at close to channel capacity requires coding. Some systems using modern codes, such as “Turbo codes” can operate within a fraction of a dB of capacity.