Assignment 3 - Solutions

1. The start flag, 01111110, is transmitted first.

   The ASCII code for character '?' is 0x3F or 0011 1111 but a zero is inserted after the fifth '1' bit so 0011 11011 is transmitted.

   The ASCII code for character ' ' is 0x7E or 0111 1110 but a zero is inserted after the fifth '1' bit so 0111 11010 is transmitted.

   The end flag, 01111110, is transmitted last.

   Note that escaping sequences of 6 bits requires inserting a '0' after 5 bits, not after 6 bits. This is what allows the receiver to distinguish the start and end flags from the data.

2. A 16-QAM signal transmits $\log_2(16) = 4$ bits per symbol. At 2400 symbols per second the bit rate is $4 \times 2400 = 9600$bps.

   For there to be no ISI the minimum-bandwidth filter is a "brick-wall" filter with a bandwidth of $f_s/2 = 1200$ Hz. If a raised-cosine filter with excess bandwidth of $\alpha = 1$ is used the bandwidth is $1 + \alpha \times f_s/2 = (1+1)/2 f_s = f_s = 2400$ Hz.

3. The I and Q outputs are

   $Q(t) = \cos(\omega_c t + \phi)\sin(\omega_c t)$,

   and

   $I(t) = \cos(\omega_c t + \phi)\cos(\omega_c t)$.

   Using

   $A = \omega_c$

   $B = \omega_c t + \phi$

   and the two identities provided,

   $Q(t) = \frac{1}{2}[\sin(2\omega_c + \phi) + \sin(-\phi)]$.

   and

   $I(t) = \frac{1}{2}[\cos(2\omega_c + \phi) + \cos(-\phi)]$.

   To recover the baseband value $cos(-\phi)$ and $-sin(\phi)$ we need a low-pass filter that passes the baseband signal but attenuates (filters out) the components at frequency $2\omega_c$. An example would be a low-pass filter that passes frequencies below f_c.

   Note that by using $-\sin(\omega_c t)$ as the LO signal for Q we would recover ϕ instead of $-\phi$.

4. We can recover $\log_2(8) = 3$ bits per symbol when using an 8-level ASK signal. One gray-coded mapping, from voltage levels to bits is shown below. The second mapping can be derived by any means that retains the property of gray coding such as reversing the order of the bits as shown.

voltage	bits	bits
-7	000	000
-5	001	100
-3	011	110
-1	010	010
1	110	011
3	111	111
5	101	101
7	100	001

5. A GMSK signal is an MSK signal. MSK signals have a frequency deviation that is one-half the bit rate. For a bit rate of 200 kHz the frequency deviation is $200/2 = 100$ kHz. If the lower of the two frequencies used by the frequency modulator is 10 MHz, the higher frequency must be $10 + 0.1 = 10.1$ MHz.

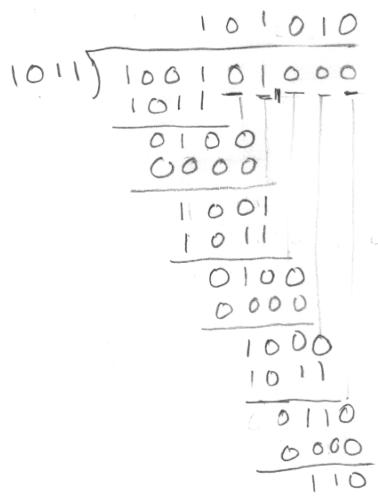
6. For the 16-QAM 802.11n constellation given in lecture 9 a symbol with a phase of -135 degrees could correspond to the point at either $-1-j$ or $-3-3j$. If the constellation had been scaled for a maximum amplitude of 1, each amplitude would have been scaled by $1/\sqrt{3^2+3^2} = 1/(3\sqrt{2})$. An amplitude of 0.33 volts would correspond to an amplitude in the unscaled constellation of $\sqrt{2}$ so the point must be at $-1-j$ and the bits must be 0101.

7. The duration of a 1024-byte (8192 bit) frame transmitted over a 100 Mb/s is $8192/(100 \times 10^6) \approx 82\mu s$. This is much less than the specified 50 ms propagation delay.

   Stop-and-wait ARQ would result in a 50 ms delay between each 82 μs packets and would result in a throughput of about $8192/0.05 = 163$ kb/s so it would not be a good choice.

   If there is only one error per day then go-back-N and selective repeat ARQ protocols would have approximately the same throughput but go-back-N ARQ would be simpler to implement so it would be the best choice.

8. To compute the CRC to be appended to a data frame consisting of the bits 100101 (in that order) with a CRC generator polynomial of $x^3 + x + 1$ (1011) we would multiply by x^3 to append 3 zeros bits (highest order of the generator polynomial) and compute the CRC as the remainder of dividing the message polynomial by the generator polynomial as shown below:

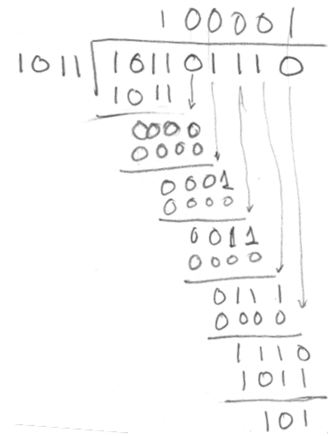
   

   The remainder, 110, is appended to the message. The bits transmitted are: 100101 110.

   Since there are $k = 6$ data bits and $n - k = 3$ parity bits, there are $n = 9$ total bits and this could be described as a (9,6) block code.

   If transmission over a channel resulted in an error in the third bit, the bits 101101 110 would be received. The Hamming distance between the transmitted and received codewords is 1 (1 bit difference).

   If the received bits (with the error) were divided by the generator polynomial the remainder can be computed as:

   The remainder of 101 rather than zero indicates an error in the received data.