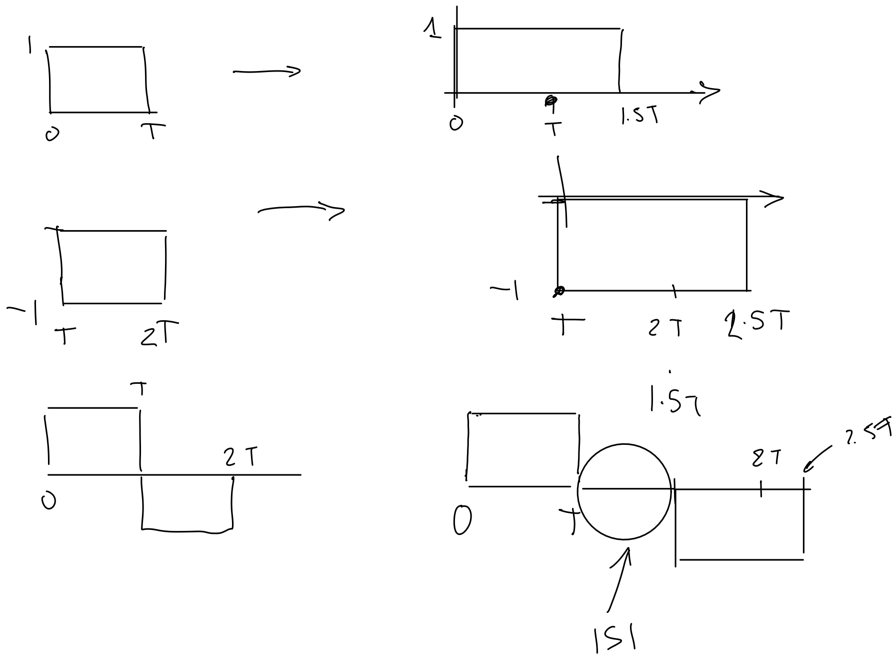
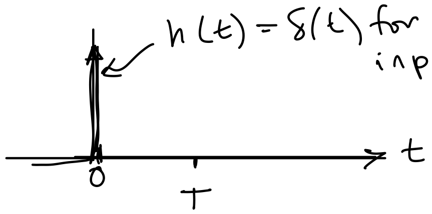
Data Transmission over Bandlimited Channels

Exercise 1: Draw a square pulse of duration T and amplitude 1. Draw the output if the channel stretches pulses to a duration of $1.5T$. Draw the output for an input pulse of the opposite polarity. Use the principle of superposition to draw the output of the channel for a positive input pulse followed by a negative input pulse. Has the signal been distorted?



Exercise 2: What is the impulse response of a channel that does not alter its input? Does this impulse response meet the Nyquist condition? Will it result in ISI?

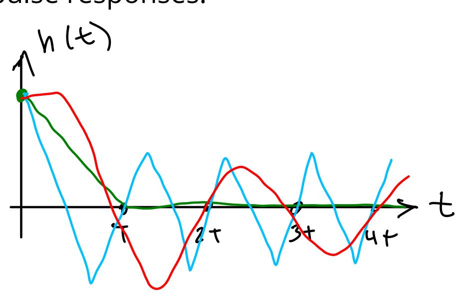
$h(t) = \delta(t)$ for channel that doesn't change its input.



impulse response $(\delta(t))$ is zero for all $t \neq 0$.

$\therefore$ no ISI.

Exercise 3: Draw the impulse response of a channel that meets the Nyquist condition but is composed of straight lines. Note that there are many such impulse responses.



Exercise 4: What causes the $\mathrm{sinc}()$ function to have periodic zero-crossings? What causes the amplitude to decay?

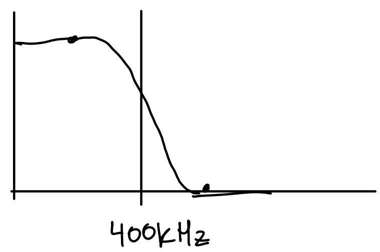
$$\frac{\sin(\pi t/T)}{\pi t/T}$$

periodic due to $\sin(\cdot)$

decays due to $\frac{1}{t}$

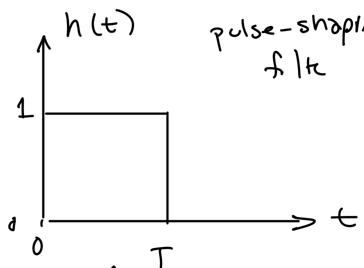
Exercise 5: Draw the magnitude of a raised-cosine transfer function that would allow transmission of impulses at a rate of 800 kHz with no interference between the impulses.

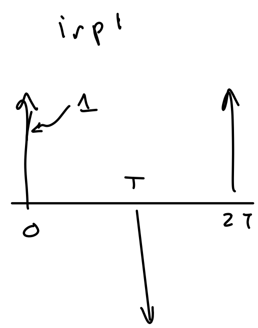
800kHz.



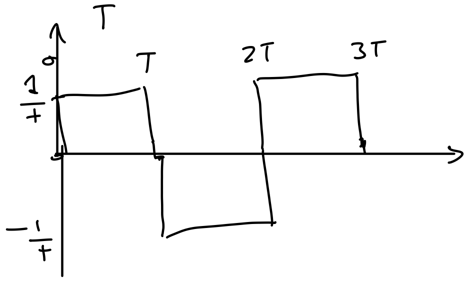
Exercise 6: Draw the impulse response of a filter than converts input impulses to pulses of duration T? Draw the signal after the pulse-shaping filter in the diagram above.

pulse-shaping filter

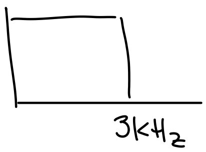




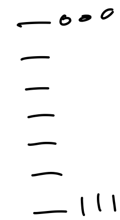
output



Exercise 7: A "brickwall" channel has a 3 kHz bandwidth and meets the Nyquist non-ISI conditions. How many levels are required to transmit 24 kb/s over this channel using multi-level signalling?



24kb/S



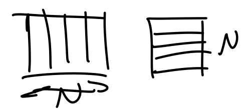
6000 symbols/S,

need 4 bits $\left(\frac{24000 b/s}{6000\ sym/s} = 4\ b/sym.\right)$ per symbol.

need $\log_2(n) = 4$

$n = 2^4 = 16$ levels.

Exercise 8: The 802.11g WLAN standard uses OFDM with a sampling rate of 20 MHz, with $N = 64$ and guard interval of $0.8\mu s$. What is the total duration of each OFDM block, including the guard interval? How long is the guard time?



$N = 64$ samples at $\underline{20\text{ MHz}}$ $+\ 0.8\mu s.$

$\boxed{850n?2}$

$\frac{1}{20\times10^6} = 50ns.$

$64 \cdot 50 = 3.2\mu s + 0.8\mu s = \underline{\underline{4\mu s.}}$

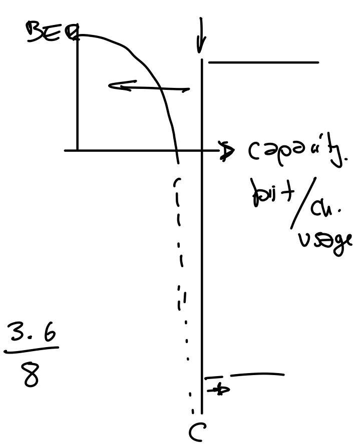
BSC

Exercise 9: What is capacity of a binary channel with a BER of $\frac{1}{8}$ (assuming the same BER for 0's and 1's)? *Hint:* $\log_2(\frac{7}{8}) \approx -0.2$.

$$C = 1 - (-p\log_2 p - (1-p)\log_2(1-p))$$

$$C = 1 - \left(-\frac{1}{8}\overbrace{\log_2\frac{1}{8}}^{-3} - \left(1-\frac{1}{8}\right)\log_2\left(1-\frac{1}{8}\right)\right)$$

$$= 1 - \left(\frac{3}{8} + \frac{1.4}{8}\right) = 1 - \frac{4.4}{8} = \frac{3.6}{8}$$



Exercise 10: What is the channel capacity of a 4 kHz channel with an SNR of 30dB?

$$30 = 10 \log_{10}\left(\frac{S}{N}\right).$$

$$\frac{S}{N} = 10^{\frac{30}{10}} = 1000$$

$$C = B \log_2\left(1 + \frac{S}{N}\right)$$

$$= 4000 \log_2(1 + 1000).$$

$$\approx 4000 \cdot 10$$

$$= 40{,}000.$$

Exercise 11: Can we use compression to transmit information faster than the (Shannon) capacity of a channel? To transmit data faster than capacity? Explain.

information rate → rate after best possible compression

data → can exceed Shannon capacity (e.g. w/ coding w/ errors).

Exercise 12: What do the Nyquist no-ISI criteria and the Shannon Capacity Theorem limit? What channel parameters determine these limits?

Nyquist	Shannon.
limits symbol rate.	information rate.
– bandwidth. (assuming symmetry). or impulse response	(joint probability distribution) BSC → P (BER) AWGN → S/N & B