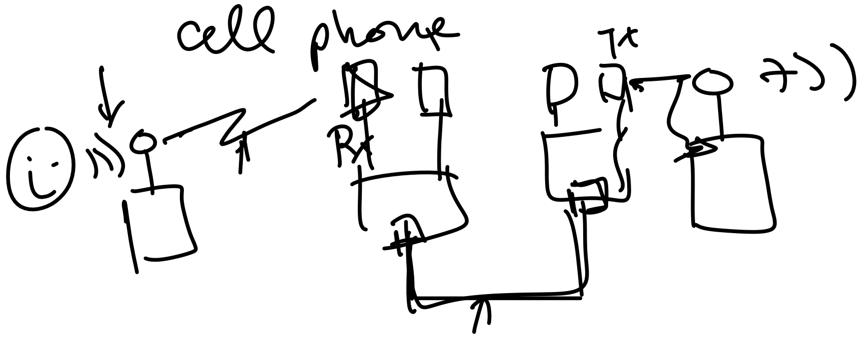
Introduction to Digital Communication

Exercise 1: Give an example of a communication network. What are the information source and sink? What channels does it operate over? What transmitters and receivers do you think are used on each channel?



Exercise 2: Speech is intelligible even if only frequencies below about 4 kHz are transmitted. What is the minimum sampling rate that should be used to sample speech if we first remove frequencies above 4 kHz?

$$f_s \geq 2 \times 4\,\text{kHz} = 8\,\text{kHz}$$

Exercise 3: A signal-to-noise power ratio of about 48 dB is considered "toll quality" (the SNR conventional telephone networks provide). How many bits per sample are required to obtain a quantization SNR equivalent to "toll quality" speech?

$$\frac{S}{N}(\text{dB}) = 6B = 48\,\text{dB} \qquad \frac{S}{N} = 20$$

$$B = 8 \text{ bits}$$

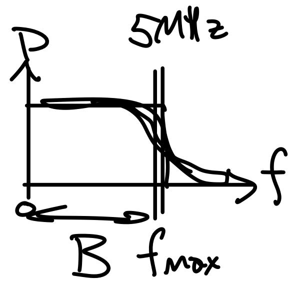
$$\text{dB} = 10 \log_{10} \frac{P_1}{P_2}$$

$$= 20 \log_{10} \left(\frac{V_1}{V_2}\right)$$

$\cancel{\ln x}$ $\cancel{\text{not}}$

$$\frac{P_1}{P_2} = \frac{V_1^2 / \cancel{R}}{V_2^2 / \cancel{R}}$$

Exercise 4: What if the signal was a video signal with a 5 MHz bandwidth and required a quantization SNR of 40 dB?



$$f_s = 2 \cdot f_{max} = 2 \cdot 5\,\text{MHz} = 10\,\text{MHz}$$

$$\text{SNR(dB)} = 6B = 40\,\text{dB}$$

$$B = 6\tfrac{2}{3}\ \text{bits}$$

(use 7 bits).

Exercise 5: How many bits per second need to be transmitted in these two examples?

$$8{,}000\,\frac{\text{samples}}{\text{s}} \cdot 8\,\frac{\text{bits}}{\text{sample}} = 64\ \text{kbps}$$

$$5\ \text{M}\frac{\text{samples}}{\text{s}} \cdot 7\,\frac{\text{bits}}{\text{sample}} = 35\ \text{Mb/s}$$

Exercise 6: Write the sequence of bits that would be transmitted if the 16-bit value 525 was transmitted with the bytes in little-endian order and the bits lsb-first. Write the sequence of bits that would be transmitted in "network order" and the bits msb-first.

this order

$$\underbrace{0000\ 0010}_{\text{MSB}}\ \underbrace{0000\ 1101}_{\text{LSB}}$$

0000 1101 0000 0010

1011 0000 0100 0000

time

Exercise 7: We observe a source that outputs letters. Out of 10,000 letters 1200 were 'E'. What would be a reasonable estimate of the probability of the letter 'E'?

$$\frac{1200}{10000} = 12\%$$

Exercise 8: A source generates four different messages. The first three have probabilities 0.125, 0.125, 0.25. What is the probability of the fourth message? How much information is transmitted by each message? What is the entropy of the source? What is the average information rate if 100 messages are generated every second? What if there were four equally-likely messages?

$$P_i = \frac{1}{8}, \frac{1}{8}, \frac{1}{4}, \frac{1}{2} \qquad \left(1 - \left(\frac{1}{8} + \frac{1}{8} + \frac{1}{4}\right) = \frac{1}{2}\right).$$

$P_0 \quad P_1 \quad P_2 \quad P_3$

$$I_0 = -\log_2(P_0) = 3$$

$$I_1 = 3$$

$$I_2 = 2$$

$$I_3 = 1$$

$$H = 3 \cdot \frac{1}{8} + 3 \cdot \frac{1}{8} + 2 \cdot \frac{1}{4} + 1 \cdot \frac{1}{2}$$

$$= I_0 \cdot P_0 + I_1 \cdot P_1 + I_2 \cdot P_2 + I_3 \cdot P_3 \equiv \sum_i I_i \cdot P_i$$

$= 1.75$ bits/message.

$$\frac{\frac{\text{bit}}{\text{msg}}}{\frac{\text{msg}}{\text{s}}} = \frac{\text{bit}}{\text{msg}} \cdot \frac{\text{s}}{\text{msg}}$$

$$100 \ \frac{\text{messages}}{\text{second.}}$$

infor. rate $= 1.75 \times 100 \ \frac{\text{bits}}{\text{msg}} \cdot \frac{\text{msg}}{\text{sec.}}$

$= 175$ bits/second

$P_i = \frac{1}{4} \qquad I_i = \underline{2 \text{ bits}} \qquad H = \frac{1}{4} \cdot 2 + \frac{1}{4} \cdot 2 + \frac{1}{4} \cdot 2 + \frac{1}{4} \cdot 2 = 2$

$= 200$ bits/s at 100 msg/s.

Exercise 9: How long will it take to transfer 1 MByte at a rate of 10 kb/s?

838 s

$$\frac{8 \times 1 \times 10^6}{10 \times 10^3} \quad \frac{b}{b/s} = 800\,s.$$

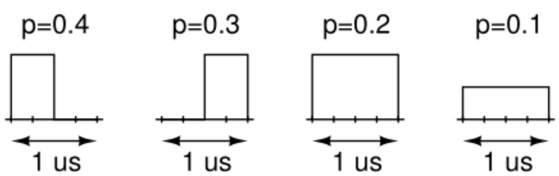
$$2^{20} = 2^{10} \cdot 2^{10}$$

$$\begin{array}{r} 1024 \\ \cdot\ 1024 \\ \hline \end{array}$$

$$\frac{8 \times 10^3}{10}$$

or. 838.8 s (if $M \equiv 2^{20}$).

$$= \frac{8 \cdot 1 \times 2^{20}}{10 \times 10^3}$$



$$\log_2(2^n) \equiv n$$

$$\log_{10}(10^x) = x$$

Exercise 10: A communication system transmits one of the symbols above each microsecond. The probability of each symbol being transmitted is given above each symbol. What are the bit rate, the symbol rate, and the information rate?

$$\text{bit rate} = \frac{\log_2(4)}{1 \times 10^{-6}} = 2\ Mb/s.$$

$$\text{symbol rate} = \frac{1}{1 \times 10^{-6}} = 1\ M\ \text{symbol}/s.$$

information rate (bits/msg)

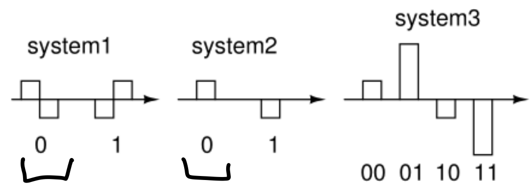
$$= -\big(0.4 \cdot \log_2(0.4) + 0.3 \log_2(0.3) + 0.2 \log_2(0.2) + 0.1 \log_2(0.1)\big)$$

$$= \downarrow$$

$$= \quad \times 1 \times 10^6 = \text{information (b/s)}.$$

Exercise 11: Another system, as shown above, encodes each bit using two pulses of opposite polarity (H-L for 0 and L-H for 1). A second system encodes bits using one pulse per bit (H for 0 and L for 1). A third system encodes two bits per pulse by using four different pulse levels (-3V for 00, -1V for 01, +1V for 10 and +3V for 11). How many different symbols are used by each system? ~~What are the symbol rates?~~ Assuming each system transmits at 1000 bits per second, what are the symbol rates in each case? Assuming each symbol is equally likely, what are the information rates?

[message ≡ symbol]



$I = \log_2\left(\frac{1}{4}\right)$

$= 2$ bits/symbol

assuming: 1000 bits/s

1ms 1ms 2ms

	system1	system2	system3
# symbols	2	2	4
symbol rate	1kHz	1kHz	500Hz
information rate:	$1 \cdot 1\text{kHz} = 1000$	$1 \frac{\text{bit}}{\text{sym}} \cdot 1000 \frac{\text{sym}}{\text{s}} = 1000 \frac{\text{bit}}{\text{sec}}$	$2 \text{ bits/sym} \cdot 500 \frac{\text{sym}}{\text{s}} = 1000$ bits

Exercise 12: You receive 1 million frames, each of which contains 100 bits. By comparing the received frames to the transmitted ones you find that 56 frames had errors. Of these, 40 frames had one bit in error, 15 had two bit errors and one had three errors. What was the FER? The BER?

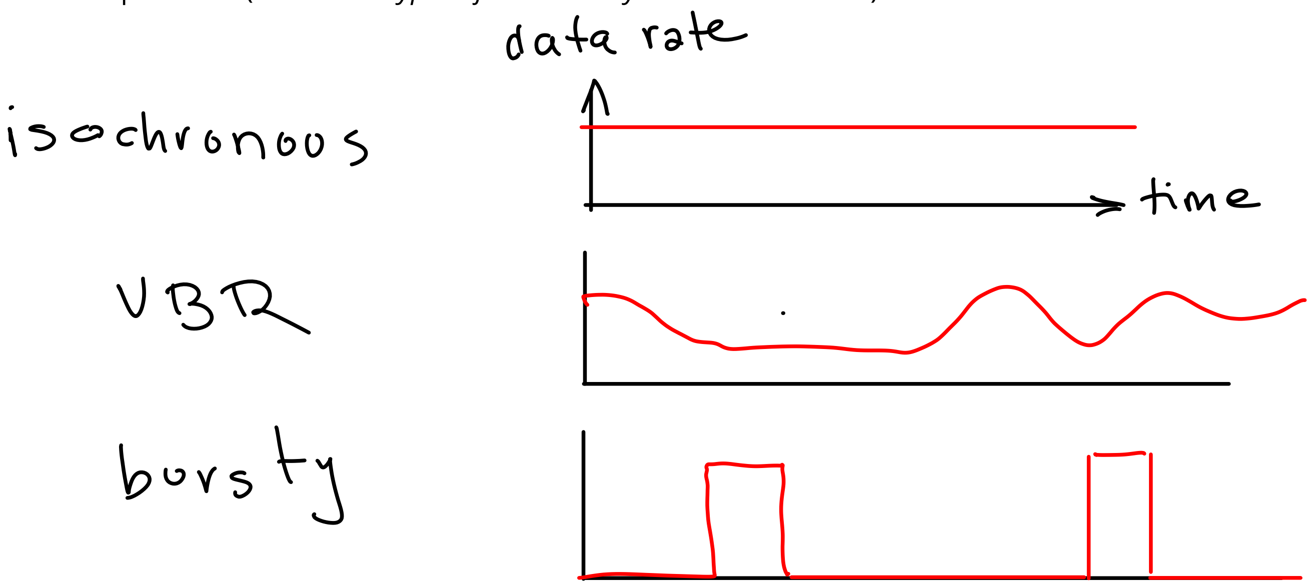
$$FER = \frac{56}{1 \times 10^6} = 56 \times 10^{-6} = 5.6 \times 10^{-5}$$

$$BER = \frac{40 \times 1 + 15 \times 2 + 1 \times 3}{100 \times 1 \times 10^6} = \frac{73}{100 \times 10^6} = 7.3 \times 10^{-7}$$

~~**Exercise 13**: A system transmits data at an (instantaneous) rate of 1 Mb/s in frames of 256 bytes. 200 of these bytes are data and the rest are overhead. The time available for transmission over the channel is shared equally between four users. A 200 μs gap must be left between each packet. What throughput does each user see? Now assume 10% of the frames are lost due to errors. What is the new throughput per user?~~

not done

Exercise 14: Plot some sample data rate versus time curves for these three types of sources. Can you think of some characteristics of a video source that might result in a variable bit rate when it is compressed? (*Hint: what types of redundancy are there in video?*)



Exercise 15: For each of the following communication systems identify the tolerance it is likely to have to errors and delay: a phone call between two people, "texting", downloading a computer program, streaming a video over a computer network. What do you think might be the maximum tolerable delay for each?

T = tolerant
NT = not "

	delay	errors	
phone call	NT	T	< 100ms
texting	T	T	10's s.
program	T	NT	10's min.
streaming	?	?	1's s.