Solutions to Final Exam

There were two versions of most questions. The values and the answers for both versions are given below.

Question 1

(a) A data source outputs one of eight equally-likely messages every millisecond. What is the *information* rate of this source in units of bits per second?

(a) Could this information be transmitted with an arbitrarily low error rate over an AWGN channel that has a 250 Hz (or 350 Hz) bandwidth if the signal power is 1 W and the noise power is 1 mW? Why or why not?

Answers

(a) For eight equally-likely messages each has a probability of $P(i) = 1/8$. The entropy of the source is $-\log_2(P(i)) = 3$ bits/messsage and the information rate is: $\frac{3 \text{ bits/message}}{0.001 \text{ seconds/message}} = \boxed{3000 \text{ bps}}$.

(a) The capacity of an AWGN channel is given by:

$$C = B\log_2(1 + S/N)$$

For a bandwidth of 250 Hz:

$$\begin{aligned} C &= 250\log_2\left(1 + \frac{1}{0.001}\right) \\ &\approx 2500 \text{ bps} \end{aligned}$$

Therefore $\boxed{\text{No}}$, this information could not be transmitted with an arbitrarily low error rate because the information rate is greater than the capacity.

or, for a bandwidth of 350 Hz:

$$\begin{aligned} C &= 350\log_2\left(1 + \frac{1}{0.001}\right) \\ &\approx 3500 \text{ bps} \end{aligned}$$

Therefore $\boxed{\text{Yes}}$, this information could be transmitted with an arbitrarily low error rate because the information rate is less than the capacity.

Question 2

You use a "true RMS" DMM to measure a noise source whose probability distribution is Gaussian. On the DC setting you read 1 V and on the AC setting you read 0.75 V (or 0.666 V). What is the probability that the voltage will be negative (less than 0)?

Answers

For an RMS voltmeter the DC reading is the average signal voltage (μ) and the AC reading is the standard deviation (σ). The normalized threshold is:

$$\begin{aligned} t &= \frac{v - \mu}{\sigma} \\ &= \frac{0 - 1}{0.75} \\ &\approx -1.33 \end{aligned}$$

From the graph in Lecture 2, $P(x < v) \approx 9\%$ (or, using a calculator $P(x < v) \boxed{\approx 9.121\%}$).
For $\sigma = 0.666$:

$$\begin{aligned} t &= \frac{0 - 1}{0.666} \\ &\approx -1.5 \end{aligned}$$

From the graph in Lecture 2, $P(x < v) \approx 7\%$ (or, using a calculator $P(x < v) \boxed{\approx 6.661\%}$).

Question 3

What is the Unicode code point of the character whose UTF-8 encoding is the three bytes **`E3 94 A5`**? Give your answer as a hexadecimal value.

Answers

The most-significant nybble of the first byte, **`E`**, implies the start of a 3-byte sequence. The bits are:

```
E3 = 1110 0011
94 = 1001 0100
A5 = 1010 0101
```

Removing the appropriate number of leading 0's and 1's from each byte:

E3 = ~~1110~~ 0011

94 = ~~1001~~ 0100

A5 = ~~1010~~ 0101

and collecting the remaining bits gives: **0011 01 01 00 10 0101** which is $\boxed{\texttt{U+3525}}$.

Question 4

Draw the waveform used to transmit the 8-bit value **0xd5** (or **0x3e**) over an asynchronous serial interface at 9600 bps (or 1200 bps) with odd parity. Label the vertical axis with voltage levels that would be correct for a transmitter. Show the duration of one bit on the horizontal axis.

Answers

Converting **0xd5** to binary gives **1101 0101**. Converting **0x3e** to binary gives **0011 1110**.

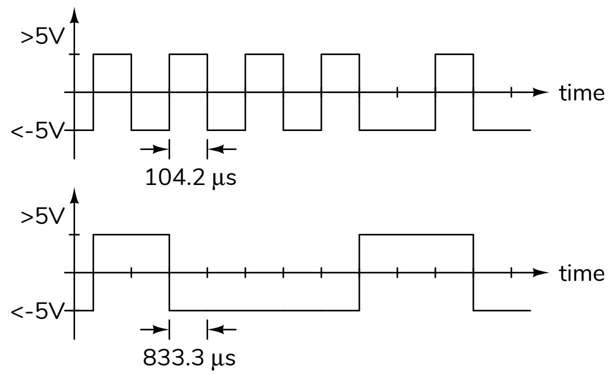
Since asynchronous serial ("RS-233") interfaces transmit bits in least-significant-bit first order, the sequence of bits to be transmitted is: **1010 1011** (or **0111 1100**).

For odd parity the number of '**1**' bits, including the parity bit, must be odd. Since the number of '**1**' bits (5) is already an odd number, the parity bit is **0**.

The bits are transmitted with a low level for a '**1**' and a high level for a '**0**'. A high start bit must be sent before the bits and a low stop bit must be added after.

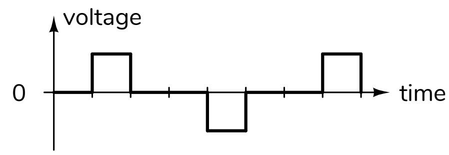
The voltage levels at the transmitter must be > 5 V for high and < -5 V for low. The duration of each bit is the inverse of the bit rate, $1/9600\text{ bps} = 104.2\,\mu\text{s}$ (or $1/1200\text{ bps} = 833.3\,\mu\text{s}$.

The resulting waveforms are:

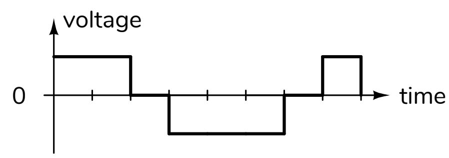


Question 5

The waveform below was encoded with an MLT-3 line code. What data was transmitted? Assume the first bit transmitted was a zero. Give your answer as a sequence of eight 1's or 0's. Tic marks indicate the bit boundaries.

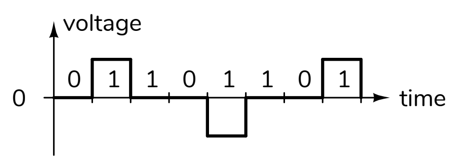


or:

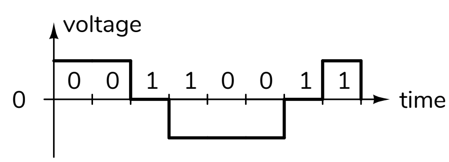


Answers

The MLT-3 line code encodes a '1' as a change in level and a '0' as no change. The sequence of bits, $\boxed{\texttt{0110 1101}}$ (or $\boxed{\texttt{0011 0011}}$), including the initial bit which was given as a zero, is drawn on the waveforms below:



or:



Question 6

A PRBS has a period of 32767 (or 1023) bits. What is the longest run of zeros in the sequence?

Answers

The period of a PRBS is $2^n - 1$. In this case $2^n - 1 = 32767$ (or $2^n - 1 = 1023$) so $n = \log_2(3267 + 1) = 15$ (or $n = \log_2(1023 + 1) = 10$) and the longest run of zeros is $n-1 = 15-1 = \boxed{\text{14 bits}}$ (or $n-1 = 10-1 = \boxed{\text{9 bits}}$).

Question 7

A message consisting of the bits `1101` is to be protected using a CRC with a generator polynomial of $x^3 + x^2 + x + 1$.

(a) What is the length of the CRC in units of bits?

(b) What is the value of a CRC computed using the simple method described in the lecture notes?

(c) What message would be transmitted (data plus CRC)?

(d) What are the values of k and n?

Answers

(a) The length of the CRC is equal to the order of the Generator polynomial ($\boxed{\text{3 bits}}$). This is also one less than the number of terms in the polynomial (3).

(b) The simple CRC is the remainder after dividing by the generator polynomial. We can do this by long division of the message multiplied by x^{n-k} ($n - k = 3$ zero bits appended):

```
1101000
1111
----
 0100
 ----
  1000
  1111
  ----
   1110
   1111
   ----
    001
```

The CRC is $\boxed{\texttt{001}}$.

(c) The message transmitted is the data with the CRC appended: $\boxed{\texttt{1101 001}}$.

Optionally, we can check the result by appending the CRC to the message and repeating the division:

```
1101001
1111
----
 0100
 ----
  1000
  1111
  ----
   1111
   1111
   ----
    000
```

(d) The number of data bits transmitted is $\boxed{k = 4}$, and the total number of bits transmitted is $\boxed{n = 7}$.

Question 8

A code has the following three six-bit codewords:

- `000000`
- `111000`
- `111111`

(a) What is the minimum distance of this code?

(b) What is the maximum number of errors that are guaranteed to be detected?

(c) What is the maximum number of errors that are guaranteed to be corrected?

(d) If the codeword `000011` is received, what codeword was most likely transmitted?

Answers

- The Hamming distances between each pair of codewords is computed in the following table:

	000000	111000	111111
000000	0	3	6
111000		0	3
111111			0

  and the minimum is $\boxed{d_{\text{min}} = 3}$.
- $d_{\text{min}} - 1 = \boxed{\text{2 errors can be detected}}$.
- $\left\lfloor \frac{d_{\text{min}}-2}{2} \right\rfloor = \boxed{\text{1 errors can be corrected}}$.
- The Hamming distances from the codeword `000011` to the three valid codewords are 2, 5 and 4 bits respectively so the first codeword $\boxed{\texttt{000000}\text{ was most likely transmitted}}$.

Question 9

A differential signal has a common-mode voltage of 100 mV (or 50 mV) and a differential voltage of −200 mV (or −100 mV). What are the voltages on the two conductors relative to ground? You may use any variable names for these two voltages.

Answers

Let the two voltages relative to ground be V_+ and V_-.

The common mode voltage is given as 100 mV and is defined as

$$V_{\text{cm}} = \frac{V_+ + V_-}{2}$$

The differential mode voltage is given as −200 mV and is defined as

$$V_{\text{diff}} = V_+ - V_-$$

We can solve for V_+ and V_- by elimination, substitution, or using a calculator.

By elimination, we multiply the equation for the common mode voltage:

$$0.5V_+ + 0.5V_- = 100$$

by 2 to obtain

$$V_+ + V_- = 200$$

and add the equation for the differential voltage to both sides

$$V_+ - V_- = -200$$

so that

$$2V_+ = 0$$

giving $\boxed{V_+ = 0\,\text{mV}}$. Substituting in either equation gives $\boxed{V_- = 200\,\text{mV}}$.

The second version of the problem has $V_{\text{cm}} = 50\,\text{mV}$ and $V_{\text{diff}} = -100\,\text{mV}$. To solve this using a calculator we write the two equations as:

$$0.5V_+ + 0.5V_- = 50$$

and

$$V_+ - V_- = -100$$

and enter the coefficients $(0.5, 0.5, 50)$ and $(1, -1, -100)$ into a calculator to obtain: $\boxed{V_+ = 0\,\text{mV}}$ and $\boxed{V_- = 100\,\text{mV}}$.

Question 10

Of the ARQ protocol(s) studied in this course, which one(s) would have high throughput over a channel with a long delay relative to the packet duration and a very low error rate?

Answers

Go-back-N and selective repeat have high throughput over channels with long delays. If the error rate is very low then selective repeat does not significantly increase the throughput. Thus, both $\boxed{\text{go-back-N and selective repeat}}$ would have high throughput.