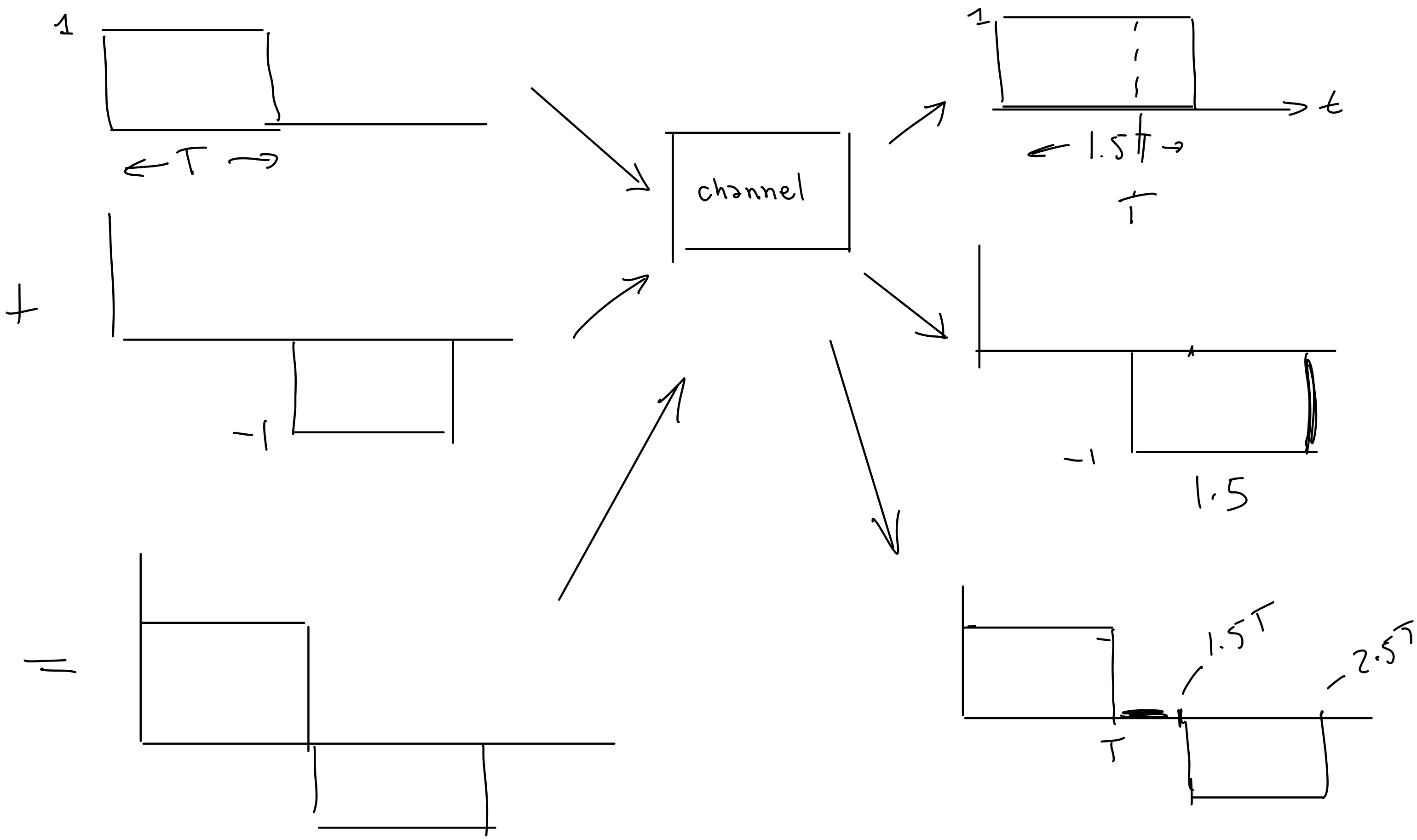
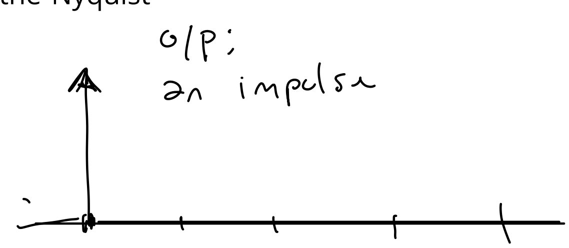
Data Transmission over Bandlimited Channels

Exercise 1: Draw a square pulse of duration T and amplitude 1. Draw the output if the channel stretches pulses to a duration of $1.5T$. Draw the output for an input pulse of the opposite polarity. Use the principle of superposition to draw the output of the channel for a positive input pulse followed by a negative input pulse. Has the signal been distorted?

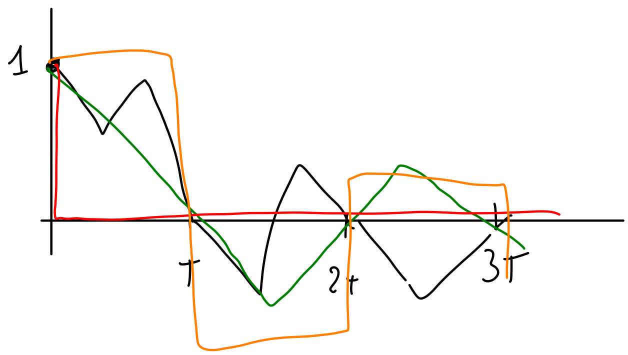


Exercise 2: What is the impulse response of a channel that does not alter its input? Does this impulse response meet the Nyquist condition? Will it result in ISI?

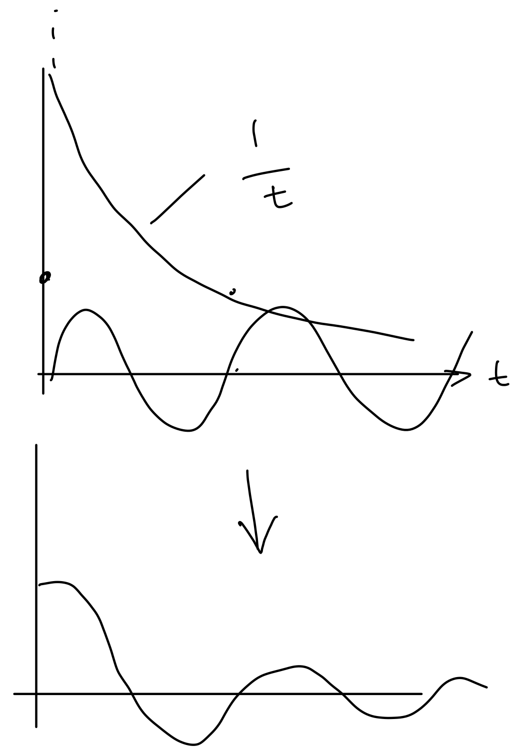


Yes, no ISI

Exercise 3: Draw the impulse response of a channel that meets the Nyquist condition but is composed of straight lines. Note that there are many such impulse responses.

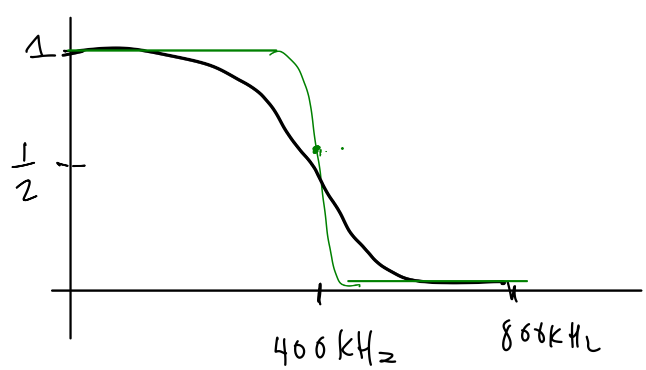


Exercise 4: What causes the sinc() function to have periodic zero-crossings? What causes the amplitude to decay?

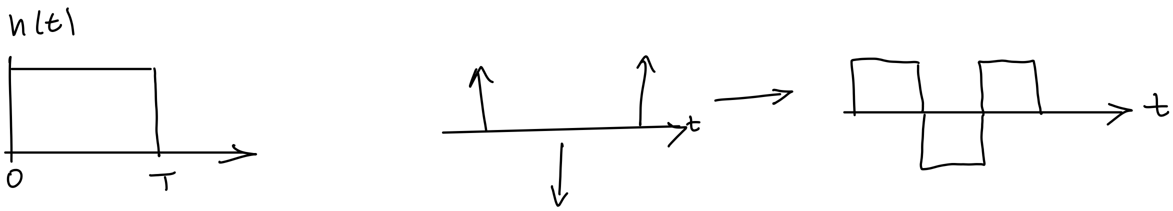


$$h(t) = \text{sinc}\left(\frac{t}{T}\right) = \frac{\sin(\pi t/T)}{\pi t/T}$$

Exercise 5: Draw the magnitude of a raised-cosine transfer function that would allow transmission of impulses at a rate of 800 kHz with no interference between the impulses.



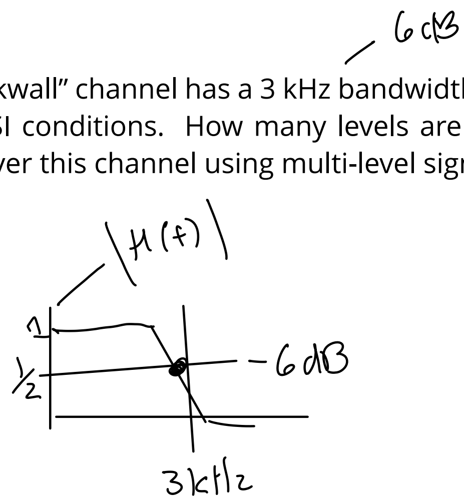
Exercise 6: Draw the impulse response of a filter than converts input impulses to pulses of duration T? Draw the signal after the pulse-shaping filter in the diagram above.



Exercise 7: A "brickwall" channel has a 3 kHz bandwidth and meets the Nyquist non-ISI conditions. How many levels are required to transmit 24 kb/s over this channel using multi-level signalling?

$-3\,dB = \frac{1}{2}$ power

$-3\,dB = \frac{1}{\sqrt{2}}$ voltage



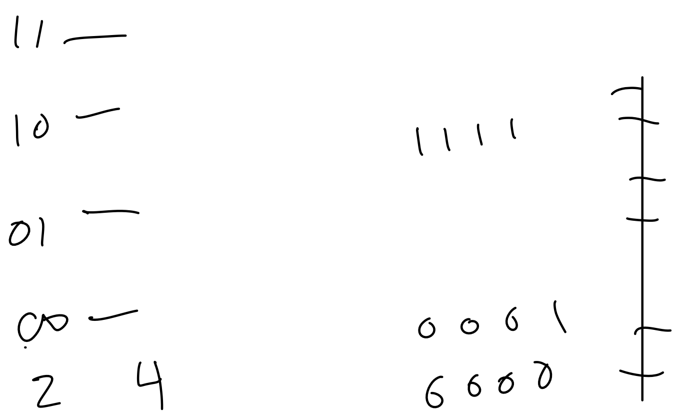
$10\log\left(\frac{P_2}{P_1}\right) = 20\log\left(\frac{V_2}{V_1}\right) = -6\,dB$

$f_{symbol} = 6\,kHz = 6000$ symbols/s

need 24 kb/s.

b/s ✓ 24000 — b/sym — sym/s.

$$\frac{24000\ b/s}{6000\ sym/s} = 4\ b/sym.$$

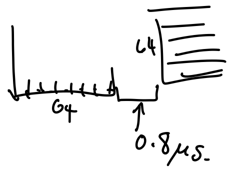


$2^4 = 16$ levels

Exercise 8: The 802.11g WLAN standard uses OFDM with a sampling rate of 20 MHz, with $N = 64$ and guard interval of $0.8\mu s$. What is the total duration of each OFDM block, including the guard interval? How long is the guard time?

$f_s = 20$ MHz $\quad T_s = \frac{1}{20 \text{ MHz}} = 50$ ns.

$N = 64$



$(Re + jIm)$

f_s complex samples

$\equiv 2 f_s$ real samples.

duration $= 64 \cdot 50\text{ns} + 0.8\mu s$

$= 3.2 + 0.8\ \mu s = 4 \mu s.$

Exercise 9: What is capacity of a binary channel with a BER of $\frac{1}{8}$ (assuming the same BER for 0's and 1's)? *Hint:* $\log_2(\frac{7}{8}) \approx -0.2$.

$$C = 1 - (-p \log_2 p - (1 - p) \log_2(1 - p))$$

$\log_2 \frac{1}{8} = -3$

$$C = 1 - \left(-\frac{1}{8} \log_2 \frac{1}{8} - \left(1 - \frac{1}{8}\right) \log_2 \left(1 - \frac{1}{8}\right)\right)$$

$$= 1 - \left(-\frac{1}{8}(-3) - \left(\frac{7}{8}\right)(-0.2)\right)$$

$$= 1 - \left(\frac{3}{8} + \frac{1.4}{8}\right)$$

$$= 1 - \left(\frac{4.4}{8}\right) = \frac{3.6}{8}$$

$\frac{\text{bits (information)}}{\text{message.}}$

Exercise 10: What is the channel capacity of a 4 kHz channel with an SNR of 30dB?

$$B = 4000$$

$$\frac{S}{N} = 10^{\frac{30}{10}} = 1000$$

$$C = B \log_2\left(1 + \frac{S}{N}\right)$$

$$= 4000 \log_2(1001)$$

$$\approx 40{,}000 \ \frac{\text{bits (information)}}{\text{s}}$$

Exercise 11: Can we use compression to transmit information faster than the (Shannon) capacity of a channel? To transmit data faster than capacity? Explain.

NO

YES

Exercise 12: What do the Nyquist no-ISI criteria and the Shannon Capacity Theorem limit? What channel parameters determine these limits?

Nyquist – symbol rate w/o ISI

Shannon – info. rate w/o errors.