Noise

This chapter describes noise and how to compute noise voltage probabilities for Gaussian noise.
After this chapter you should be able to: compute SNR and compute the probability that a Gaussian source will lie within a certain range.

Noise and SNR

Noise is a random (meaning unpredictable) signal that is added to the desired signal by the channel or by the receiver.

Sources of noise include: thermal noise present in any resistor at temperatures above 0 K, "shot" noise generated by semiconductor devices, impulse noise caused by switching currents in equipment such as motors and lighting, noise from lightning and from the sun.

Noise is the phenomenon that ultimately limits the performance of any communication system. Noise may cause errors in digital communication system or degrade the quality of an analog signal.

An important metric is the signal-to-noise ratio (SNR) which is the ratio of signal power to noise power.

Exercise 1: A zero-mean sinusoidal signal is being transmitted over a noisy telephone channel. The voltage of the signal is measured with an oscilloscope and is found to have a peak voltage of 1V.

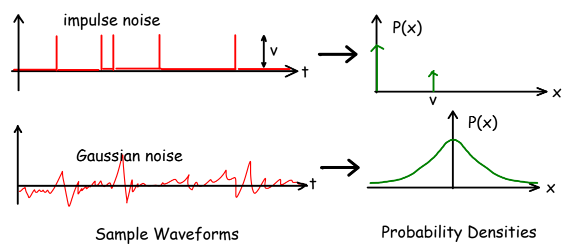
Nearby machinery induces noise into the line. The voltage of this noise signal is measured with an RMS voltmeter as 100 mVrms. The line is connected to ("terminated with") a 600Ω load.

Why was an RMS voltmeter used to measure the noise? What signal power is dissipated in the load? What noise power? What is the SNR? Does the SNR depend on the load impedance?

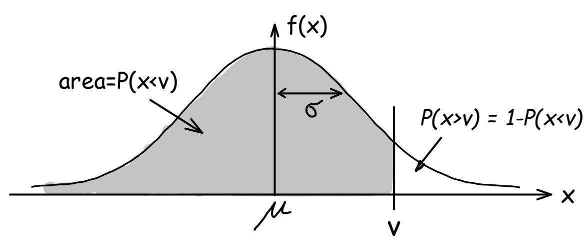
Gaussian Probability Distribution

To determine the effect of noise on a communication system we need to know the probabilities of different voltage levels. Two noise sources may have the same power but have different effects on the error rate. For example, if the noise never exceeds a voltage that would cause an error then the noise has no effect.

A probability distribution describes the likelihood that a random variable (e.g. noise) will have a certain value. For discrete random this distribution can be drawn as bar graph (e.g. a histogram) but for continuous variables the probability distribution is a continuous curve giving the "probability density" (pdf) of outcomes around that voltage. The probability that the noise will be within a range of values is the integral (area) under the curve between those values.



Signals that result from the sum of many small independent events have a probability distribution known as a Gaussian (or "normal") distribution. In communication systems this usually happens due to the sum of of voltages produced by the actions of many individual photons, electrons or molecules.



The Gaussian distribution is the familiar "bell curve" distribution. The probability is a maximum at the average value and drops off to smaller probabilities at larger or smaller voltages.

A Gaussian distribution is defined by two values: the mean (μ) and the variance (σ^2). The standard deviation (σ) is the square root of the variance.

Note that the variance (and standard deviation) are independent of the mean. This is the voltage measured with AC coupling. This is different than mean-square or "AC+DC" RMS values which *do* include the average value.

To compute error rates we typically need to find the probability that the voltage of a Gaussian noise

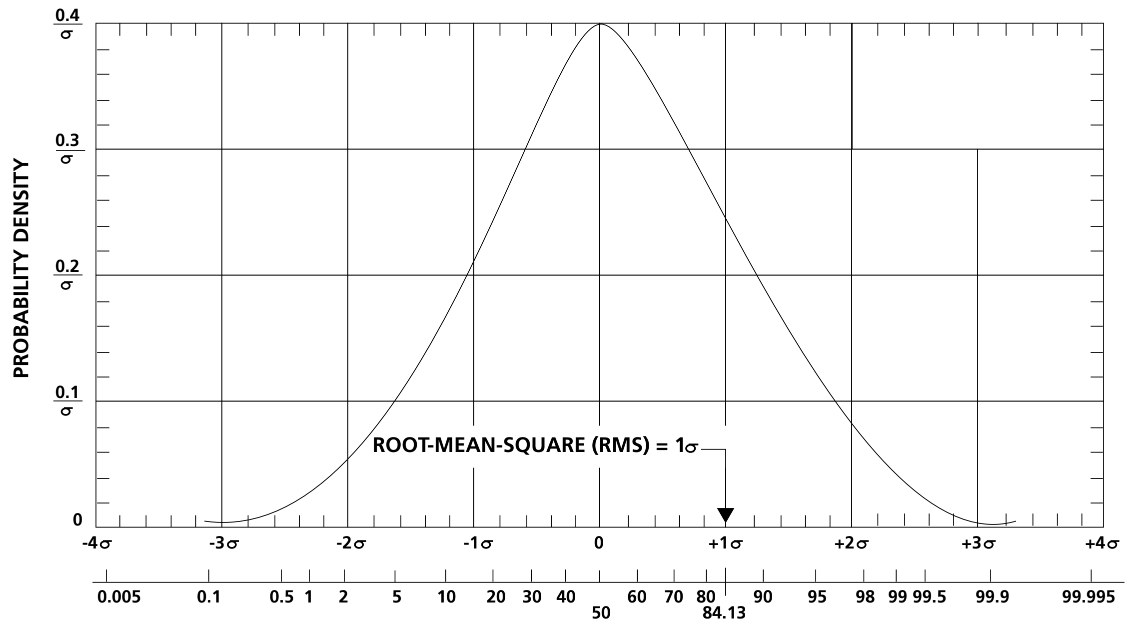


Figure 1. Gaussian Voltage Distribution

Figure 1: Gaussian density function and values of the cumulative distribution (from NoiseCom application note "Noise Basics").

signal will fall in a certain range of values. If the noise signal x has a DC (mean) value μ and a RMS *AC* (zero-mean) voltage σ then the probability (fraction of time) that the noise voltage is less than v, $P(x < v)$, is given by the Gaussian (Normal) cumulative distribution function (CDF). This is the area under the Gaussian distribution curve to the left of (less than) the value v.

Exercise 2: Would you use AC or DC coupling to measure: (a) μ, (b) σ? Would you be measuring the average or RMS power in each case?

The plot in Figure 1 shows the shape of the Gaussian density function and also gives the cumulative probabilities along the second (lower) x-axis.

To find the probability that the voltage is greater than v we can use the fact that the sum of all probabilities is 1. Thus $P(x > v) = 1 - P(x \leq v)$.

To compute $P(x < v)$ we first compute a normalized value, t by subtracting the mean, μ from v, and dividing by the standard deviation, σ, of the distribution:

$$t = \frac{v - \mu}{\sigma}$$

Exercise 3: What are the units of t?

Then we can look up t in tables or curves of the normalized Gaussian CDF to find $P(x < v)$. Some calculators will compute this ($P()$ function) or you can use the figure above.

Exercise 4: The output of a noise source has a zero-mean Gaussian (normally) distributed output voltage. The (rms) output voltage is 100 mV. What fraction of the time does the output voltage exceed 200 mV? 300 mV? *Hint: the standard deviation (σ) of a zero-mean signal is the same as its RMS voltage.*

To compute the probability that a signal will be within a range of values (e.g. between a and b) we can compute the probability that it will be below the upper limit (b) and subtract the probability that it will be below the lower limit (a):

$$P(a < x < b) = P(x < b) - P(x < a)$$

Exercise 5: Mark examples of a and b on a Gaussian pdf and the three probabilities in the equation above.

Power of Sums of Signals

The power of the sum of two signals is not, in general, the sum of their powers. For example, the sum of a signal $x(t)$ and $-x(t)$ is zero and has zero power even though each signal independently could have a non-zero power.

However, in communication systems two signals are often independent[1] and one or both have a mean of zero (e.g. additive zero-mean noise or two independent speech signals). In this case the power of the sum is the sum of the powers.

Exercise 6: A signal $x(t)$ randomly switches between ± 1. What are the mean-square powers of $x(t)$, $y(t)$ and $x(t) + y(t)$ if: (a) $y(t) = x(t)$? (b) $y(t) = -x(t)$? (c) $y(t)$ randomly changes between ± 1 independently of $x(t)$? *Hint: work out the possible values and their probabilities.*

[1]Strictly speaking, statistically independent.