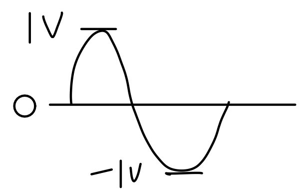
Noise

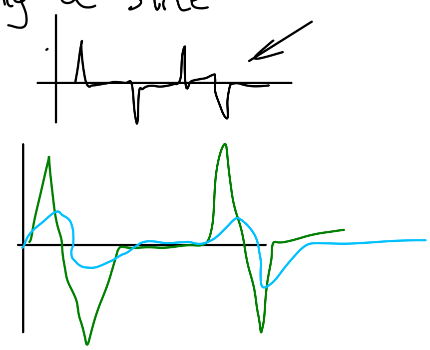
Exercise 1: A zero-mean sinusoidal signal is being transmitted over a noisy telephone channel. The voltage of the signal is measured with an oscilloscope and is found to have a peak voltage of 1V.

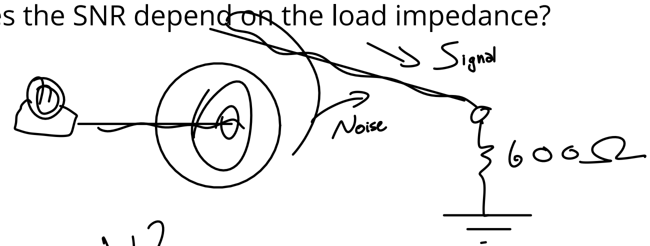
Nearby machinery induces noise into the line. The voltage of this noise signal is measured with an RMS voltmeter as 100 mVrms. The line is connected to ("terminated with") a 600Ω load.

Why was an RMS voltmeter used to measure the noise? What signal power is dissipated in the load? What noise power? What is the SNR? Does the SNR depend on the load impedance?



noise is not usually a sine wave





signal power: $S = \frac{V_{rms}^2}{R} = \frac{\left(\frac{1}{\sqrt{2}}\right)^2}{600}$ ✓

noise power: $N = \frac{V_{rms}^2}{R} = \frac{(0.1)^2}{600}$ ✓

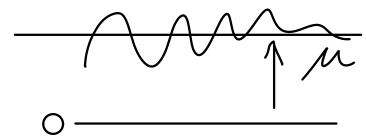
load impedance cancels

$$SNR = \frac{S}{N} = \frac{\left(\frac{1}{\sqrt{2}}\right)^2}{(0.1)^2} = \left(\frac{1}{0.1\sqrt{2}}\right)^2 = \frac{1}{.01} \cdot \frac{1}{2} = \underline{\underline{50}}$$

in dB: $10\log(50) \approx 17\text{ dB}$

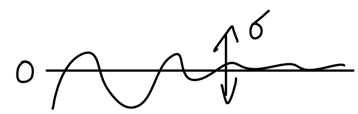
power ratio

Exercise 2: Would you use AC or DC coupling to measure: (a) μ, (b) σ? Would you be measuring the average or RMS power in each case?



(a) μ : mean: use DC coupling (average)

(b) σ : std. dev. : (rms volt.) AC coupling (RMS).



Exercise 3: What are the units of t?

unitless $\left(\frac{V}{V}\right)$

Exercise 4: The output of a noise source has a zero-mean Gaussian (normally) distributed output voltage. The (rms) output voltage is 100 mV. What fraction of the time does the output voltage exceed 200 mV? 300 mV? *Hint: the standard deviation (σ) of a zero-mean signal is the same as its RMS voltage.*

$\sigma = \sqrt{\overline{(x-\bar{x})^2}}$

$\equiv$ rms voltage

$\mu = 0$ V $\qquad \sigma = 100$ mV

$P(v > 200\text{ mV}) = 1 - P(v < 200\text{ mV})$

$$t = \frac{v-\mu}{\sigma} = \frac{200-0}{100} = 2$$

$P(v > 200\text{ mV}) = 1 - \underbrace{P(t)}_{\text{normal}} = 1 - 0.97 = 0.03$

normal CDF → area under curve to left of t

$P(v > 300\text{ mV})\ 1 - P(3) = .00135$

1.35×10^{-3}

Exercise 5:

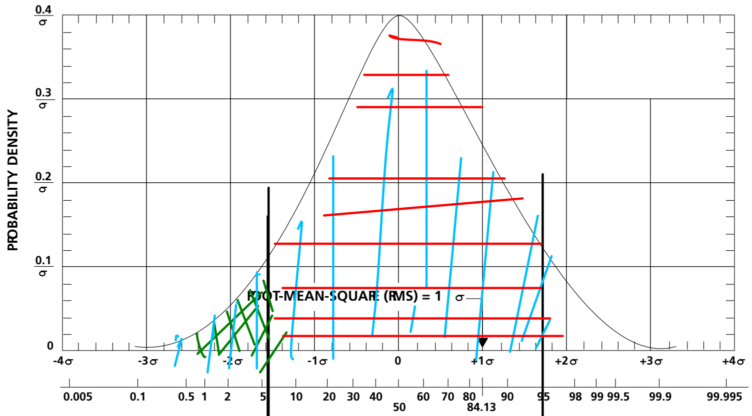


Figure 1. Gaussian Voltage Distribution

a b

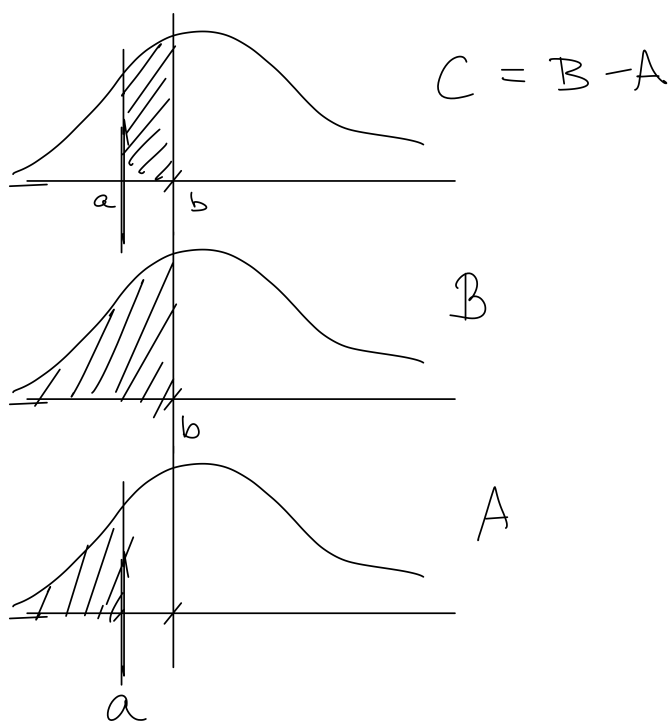
$/// \; P(x<a)$

$||| \; P(x<b)$

$\equiv \; P(a<x<b)$

$$P(a < x < b) = P(x < b) - P(x < a)$$

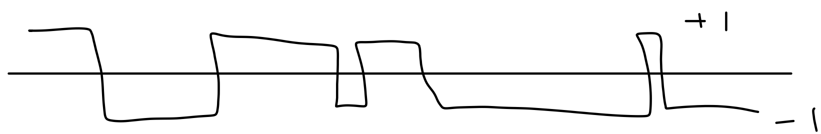
Mark examples of a and b on a Gaussian pdf and the three probabilities in the equation above.



Exercise 6: A signal $x(t)$ randomly switches between ± 1. What are the mean-square powers of $x(t)$, $y(t)$ and $x(t) + y(t)$ if: (a) $y(t) = x(t)$? (b)$y(t) = -x(t)$? (c) $y(t)$ randomly changes between ± 1 independently of $x(t)$? *Hint: work out the possible values and their probabilities.*

(not done in lecture.)

$x(t)$:



mean-square powers:

(a) $x(t)$: $\frac{1}{2}(1)^2 + \frac{1}{2}(-1)^2 = 1\ V^2$

$y(t) = x(t)$: $1\ V^2$

$x(t) + y(t)$: $\frac{1}{2}(1+1)^2 + \frac{1}{2}(-1-1)^2$

$= \frac{4}{2} + \frac{4}{2} = \underline{\underline{4}}$ $\boxed{\neq 1+1}$

(b) if $y(t) = -x(t)$:

$y(t)$: $\frac{1}{2}(-1)^2 + \frac{1}{2}(1)^2 = 1$

$x(t)+y(t)$: $\frac{1}{2}(1-1)^2 + \frac{1}{2}(-1+1)^2 = \underline{\underline{0}}\ V^2$ $\boxed{\neq 1+1}$

(c) if $y(t)$ independent of $x(t)$

possible values of $x(t)+y(t)$:

x \ y	+1	-1
+1	2	0
-1	0	-2

each has prob. of $\frac{1}{4}$

$x(t)+y(t)$: $\frac{1}{4}(2)^2 + \frac{1}{4}(0)^2 + \frac{1}{4}(0)^2 + \frac{1}{4}(-2)^2$

$= 1 + 0 + 0 + 1 = 2\ V^2$ $\boxed{= 1+1}$