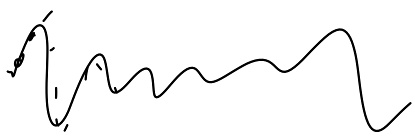
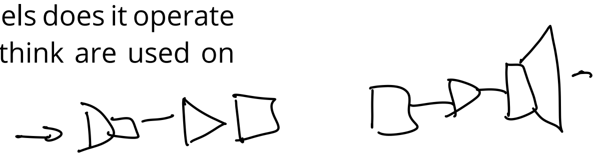
Introduction to Digital Communication

Exercise 1: Give an example of a communication network. What are the information source and sink? What channels does it operate over? What transmitters and receivers do you think are used on each channel?





channels { wireless   free space
{ optical fibre

Exercise 2: Speech is intelligible even if only frequencies below about 4 kHz are transmitted. What is the minimum sampling rate that should be used to sample speech if we first remove frequencies above 4 kHz?

$$f_s \geq 2 f_{max}$$

$$\geq 2 \cdot 4 \text{ kHz}$$

$$\geq 8 \text{ kHz}$$

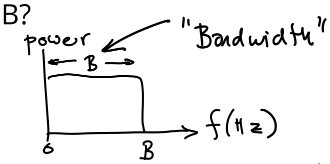
Exercise 3: A signal-to-noise power ratio of about 48 dB is considered "toll quality" (the SNR conventional telephone networks provide). How many bits per sample are required to obtain a quantization SNR equivalent to "toll quality" speech?

$$SNR_{quant} \cong 6B \quad \text{dB}$$

$$48 \text{ dB} = 6B$$

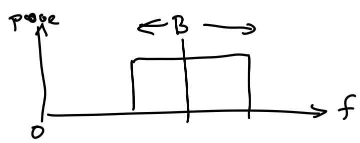
$$B = \frac{48}{6} = 8 \text{ bits}$$

Exercise 4: What if the signal was a video signal with a 5 MHz bandwidth and required a quantization SNR of 40 dB?



in 3525 "baseband" signal

$$f_s \geq 2 \cdot 5\ \text{MHz}$$
$$\geq 10\ \text{MHz}$$



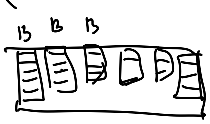
in 3521 "passband" signal

$$B\ (\text{bits}) = \frac{40\ \text{dB}}{6} = 6\tfrac{2}{3}$$

round up to 7 bits

Exercise 5: How many bits per second need to be transmitted in these two examples?

$$f_s \left(\frac{\cancel{\text{samples}}}{s}\right) \times B \left(\frac{\text{bits}}{\cancel{\text{sample}}}\right) = \frac{\text{bits}}{s}.$$



#1) $8\ \text{kHz} \cdot 8\ \text{bits/sample} = 64\ \text{kb/s}$

#2) $10\ \text{MHz} \cdot 7\ \text{bits/sampe} = 70\ \text{Mb/s}.$

1234 $\longrightarrow$

10010 $\longrightarrow$

L-to-R

R-to-L

BYTES	bits
big-endian "network order"	msb first
little endian	lsb first

Exercise 6: Write the sequence of bits that would be transmitted if the 16-bit value 525 was transmitted with the bytes in little-endian order and the bits lsb-first. Write the sequence of bits that would be transmitted in "network order" and the bits msb-first.

$525_{10} \Rightarrow 020D_{16} \Rightarrow \underbrace{0000\ 0010}_{\substack{\text{second} \\ \text{(MSB)}}}\ \underbrace{0000\ 1101}_{\substack{\text{first} \\ \text{(LSB)}}}$

$\Rightarrow$ 1011 0000 0100 0000 $\leftarrow$ little endian lsb first

$\underbrace{0000\ 0010}_{\substack{\text{first} \\ \text{(LSB)}}}\ \underbrace{0000\ 1101}_{\substack{\text{second} \\ \text{MSB)}}}$ } network order

$\Rightarrow$ 0000 0010 0000 1101

Exercise 7: How many bits would be required to uniquely identify 100,000 different characters? (Hint: $2^{16} = 65536$).

Need 17 ($2^{17} = 128k$).

Exercise 8: The Chinese character for "Rice" (the grain) is 米 with Unicode value (code point) U+7C73. What is the UTF-8 encoding for this character?

7C73

$\underbrace{0111}_{z}\ \underbrace{1100\ 01}_{y}\ \underbrace{11\ 0011}_{x}$

Table 3-6. UTF-8 Bit Distribution

Scalar Value	First Byte	Second Byte	Third Byte	Fourth Byte
00000000 0xxxxxxx	0xxxxxxx			
00000yyy yyxxxxxx	110yyyyy	10xxxxxx		
zzzzyyyy yyxxxxxx	1110zzzz	10yyyyyy	10xxxxxx	
000uuuuu zzzzyyyy yyxxxxxx	11110uuu	10uuzzzz	10yyyyyy	10xxxxxx

$\underbrace{1110\ 0111}_{E7}\quad \underbrace{10\ 110001}_{B1}\quad \underbrace{10\ 110011}_{B3}$

Exercise 9: Convert the decimal number 525 to a 16-bit (two-byte) binary number. How would you write this in hexadecimal notation?

Find the ASCII codes for the *characters* '525'. Write out the bits of the sequence that would be transmitted assuming each character is encoded in UTF-8. *Hint: the UTF-8 character code for a digit is 0x30 plus the value of the digit.*

Which of these two sequences of bits is the text format and which is the binary format? How many more bits would need to be stored or transmitted for the text format?

525 = 0x020D

0000 0010 0000 1101 ← binary

16 bits

'5', '2', '5' ⇒ 0x35 0x32 0x35 → from ASCII (or Unicode)

24 bits → text

5 2 5

24 bits for text
16 bits for binary

8 more bit

Exercise 10: We observe a source that outputs letters. Out of 10,000 letters 1200 were 'E'. What would be a reasonable estimate of the probability of the letter 'E'?

$$\frac{1200}{10000} = 12\% \equiv 0.12$$

$p_0 \quad p_1 \quad p_2 \quad p_3 = ?$

Exercise 11: A source generates four different messages. The first three have probabilities 0.125, 0.125, 0.25. What is the probability of the fourth message? How much information is transmitted by each message? What is the entropy of the source? What is the average information rate if 100 messages are generated every second? What if there were four equally-likely messages?

$$P_3 = 1 - \left(\frac{1}{8} + \frac{1}{8} + \frac{1}{4}\right) = 1 - \frac{1}{2} = \frac{1}{2}$$

$$P_0 = P_1 \qquad I_0 = I_1 = -\log_2\left(\frac{1}{8}\right) = -(-3) = 3 \text{ bits}$$

$$P_2 = \qquad -\log_2\left(\frac{1}{4}\right) = 2 \text{ bits}$$

$$P_3 = \qquad -\log_2\left(\frac{1}{2}\right) = 1 \text{ bit}$$

$$H = \bar{I} = \sum P_i I_i$$

$$= \frac{1}{8}\cdot(3) + \frac{1}{8}(3) + \frac{1}{4}\cdot(2) + \frac{1}{2}(1)$$

$$= \frac{3}{8} + \frac{3}{8} + \frac{4}{8} + \frac{4}{8} = \frac{14}{8} = 1\frac{6}{8} = 1.75 \text{ bit/msg}$$

e.g. 1000 messages has $1.75 \times 1000 = 1750$ bits of information.

If 100 msg/second $\Rightarrow 100 \frac{\cancel{msg}}{s} \cdot 1.75 \frac{bits}{\cancel{msg}} = 175$ bps

if all equally likely then $H = \sum_{i=0}^{3} \frac{1}{4} \cdot 2 = 2$ bits

$k = 10^3$

Exercise 12: How long will it take to transfer 1 MByte at a rate of $\underbrace{\text{10 kb/s}}_{10\times10^3}$?

either $\dfrac{1 \times 2^{20} \times 8}{10 \times 10^3}$ OR $\dfrac{1 \times 10^6 \text{ Byte} \times 8 \frac{\text{bits}}{\text{Byte}}}{10 \times 10^3 \text{ bits/s}}$

$= 800$ s.

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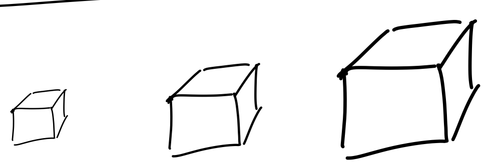

$x \to$ 1kg 2kg 3kg

$p \to$ 30% 10% $(100\% - (30+10)\%) = 60\%$

$$\text{mean} = \sum x_i p_i = 1 \cdot 0.3 + 2 \cdot 0.1 + 3 \cdot 0.6$$

$$= 0.3 + 0.2 + 1.8$$

$$= 2.3$$

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binary

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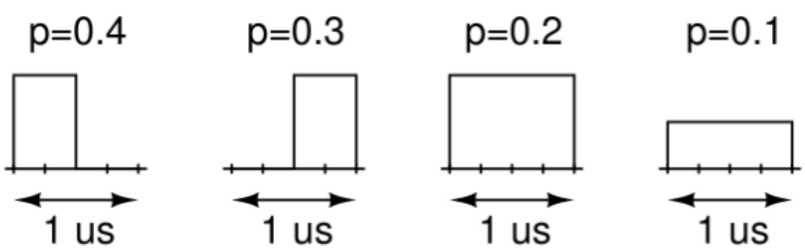
$= 800$ s.

Exercise 13: A communication system transmits one of the symbols above each microsecond. The probability of each symbol being transmitted is given above each symbol. What are the bit rate, the symbol rate, the information rate and the baud rate?



\# symbols

bit rate $= \dfrac{2 \text{ bits}}{1 \times 10^{-6} \text{ s}} = 2$ Mb/s $\left(2 \text{ bits} = \log_2 M = \log_2 4 = 2 \right)$

symbol rate $= \dfrac{1 \text{ symbol}}{1 \times 10^{-6}} = 1$ MHz

$-\log_2(P_i)$

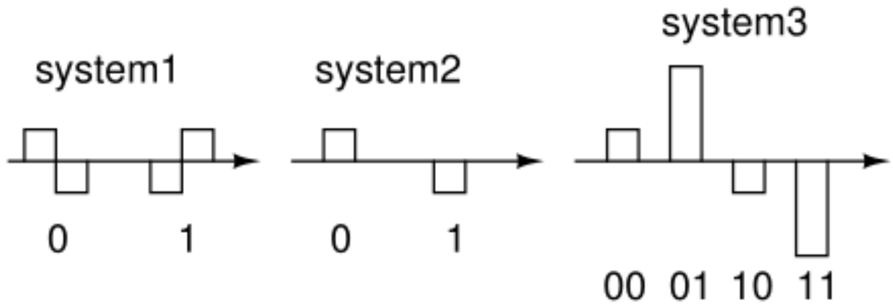
information rate $= \sum I_i P_i$

$= -\log_2(0.4) \cdot 0.4 - \log_2(0.3) \cdot 0.3 - \log_2(0.2) 0.2$

$- \log_2(0.1) 0.1 =$

baud rate $= \dfrac{1}{0.5 \times 10^{-6} \text{ s}} = 2$ MHz

Exercise 14: Another system, as shown above, encodes each bit using two pulses of opposite polarity (H-L for 0 and L-H for 1). A second system encodes bits using one pulse per bit (H for 0 and L for 1). A third system encodes two bits per pulse by using four different pulse levels (-3V for 00, -1V for 01, +1V for 10 and +3V for 11). Assuming each system transmits at 1000 bits per second, what are the baud rates in each case? How many different symbols are used by each system? What are the symbol rates? Assuming each symbol is equally likely, what are the information rates?



nomber of symbols (M)	2	2	4	(given)
bits/symbol ($\log_2 M$)	1	1	2	
bits / second	1000	1000	1000	(bitrate - given)
symbols/second	$\frac{1000}{1}$	$\frac{1000}{1}$	$\frac{1000}{2} = 500$	(symbol rate)
transitions/symbol	2	1	1	
transitions/second	2000	1000	500	(baud rate)
probabity	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{4}$	
information/symbol	$-\log_2(2^{-1}) = 1$	1	2	bits/symbol
information rate	$1000\left(\frac{1}{2}\cdot 1 + \frac{1}{2}\cdot 1\right) = 1000$		1000 $\quad 500\cdot\left(4\cdot\frac{1}{4}\cdot 2\right) = 1000$ bits/s	

Exercise 15: You receive 1 million frames, each of which contains 100 bits. By comparing the received frames to the transmitted ones you find that 56 frames had errors. Of these, 40 frames had one bit in error, 15 had two bit errors and one had three errors. What was the FER? The BER?

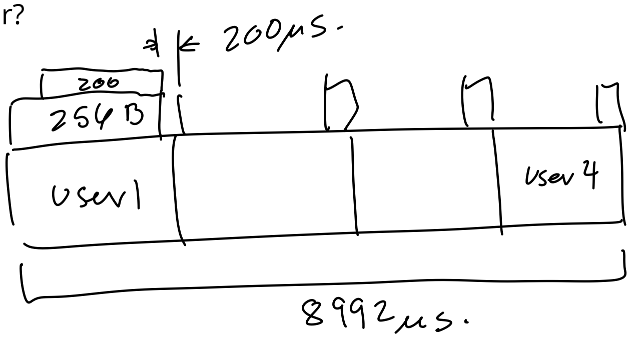
$$FER = \frac{56}{10^6} = 56 \times 10^{-6}$$

$$BER = \frac{40 \times 1 + 15 \times 2 + 1 \times 3}{100 \times 10^6} = 73 \times 10^{-8}$$

Exercise 16: A system transmits data at an (instantaneous) rate of 1 Mb/s in frames of 256 bytes. 200 of these bytes are data and the rest are overhead. The time available for transmission over the channel is shared equally between four users. A 200 μs gap must be left between each packet. What throughput does each user see? Now assume 10% of the frames are lost due to errors. What is the new throughput per user?

$200 \times 8 = 1600$ bits

$\div$



$$4\left(\frac{256\ \cancel{\text{Byte}} \cdot 8\ \frac{\cancel{\text{bit}}}{\cancel{\text{Byte}}}}{1 \times 10^6\ \cancel{\text{bits}}/s} + 200\mu s\right) \simeq 4(2048 + 200)\ \mu s$$

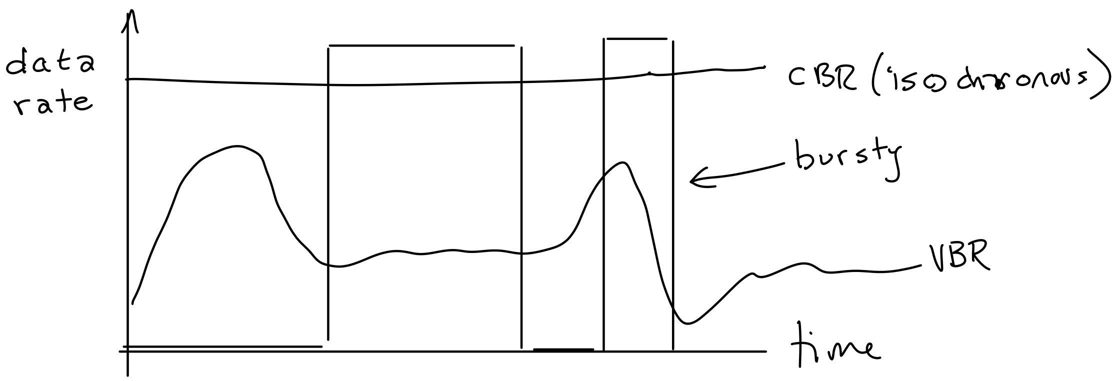
$$8992\ \mu s$$

$$\text{throughput} = \frac{1600}{8992 \times 10^{-6}} = 178\ \text{kb/s.}$$

Lets look @ 100 of these

$$\text{new throughput} = \frac{90\% \cdot 100 \cdot 1600\ \text{bits}}{100\% \cdot 100 \cdot 8992 \times 10^{-6}}$$

Exercise 17: Plot some sample data rate versus time curves for these three types of sources. Can you think of some characteristics of a video source that might result in a variable bit rate when it is compressed? (*Hint: what types of redundancy are there in video?*)



Exercise 18: For each of the following communication systems identify the tolerance it is likely to have to errors and delay: a phone call between two people, "texting", downloading a computer program, streaming a video over a computer network. What do you think might be the maximum tolerable delay for each?

	tolerance to error	tolerance to delay	
phone call	H	L	
texting	L / H	minute	
downloading	L	minutes → hours	
streaming video	H – uncompressed L – compressed	interactive streaming	< 100 ms ≈ seconds,

Exercise 19: Highlight or underline each term where it is defined in these lecture notes.