ELEX 3525 : Data Communications
2021 Fall Term

Solutions to Midterm Exam 2

Question 1

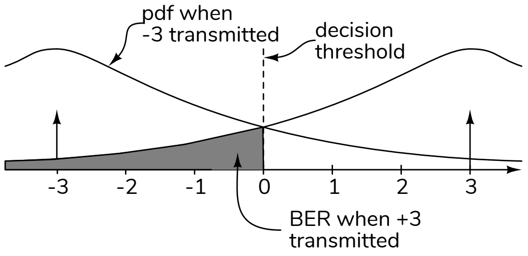
(a) A data communication system uses NRZ with voltage levels of ±3 Volts and a decision threshold at zero. What is the bit error rate if the channel adds AWGN with an RMS voltage of $1.5\ \mathrm{V_{rms}}$?

(b) What is the BER if both levels are equally probable and the channel also adds an offset voltage of +1 V?

Answer

(a) Errors will happen if the noise voltage is greater than +3 V if −3 is transmitted or is less than −3 V if +3 is transmitted. The standard deviation of the noise is given in the question as 1.5 V.

The figure below shows the probability density function (p.d.f.) of the received signal when −3 or +3 are transmitted and the area to the left of the decision threshold which is the probability of error when +3 is transmitted:

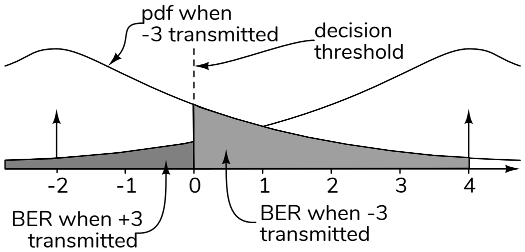


If +3 is transmitted the normalized threshold is $t = \frac{v-\mu}{\sigma} = \frac{0-3}{1.5} = -2$. The probability of the signal plus noise being less than this threshold can be obtained from a calculator or Figure 1 in Lecture 3 as 0.02275. The probability of error is the probability that the signal plus noise will be less than this threshold or ≈ 2.3%.

Similarly, if −3 is transmitted the normalized threshold is $t = \frac{v-\mu}{\sigma} = \frac{0--3}{1.5} = 2$. The probability of the signal plus noise being less than this threshold can be obtained from a calculator or Figure 1 in Lecture 3 as 0.9772. The probability of error is the probability that the signal plus noise will be more than this threshold or $1 - 0.9772 \approx 2.3\%$.

The bit error rate (BER) is the same as the symbol probability of error since with two levels we are only transmitting one bit per symbol.

(b) If the channel adds an offset voltage of +1 V the mean will be shifted by this amount. Thus the mean received signal levels will be +4 when +3 is transmitted and −2 when −3 is transmitted. The figure below shows the modified probability distributions:



When −3 is transmitted the normalized threshold is $t = \frac{v-\mu}{\sigma} = \frac{0--2}{1.5} = 1.33$. The probability of the signal plus noise being less than this threshold can be obtained from a calculator or Figure 1 in Lecture 3 as 0.9088. The probability of error is the probability that the signal plus noise will be more than this threshold or $1 - 0.9088 \approx 9.1\%$.

Similarly, when +3 is transmitted the normalized threshold is $t = \frac{v-\mu}{\sigma} = \frac{0-4}{1.5} = -2.667$. The probability of the signal plus noise being less than this threshold can be obtained from a calculator or Figure 1 in Lecture 3 as $\approx 3.8 \times 10^{-3}$. This is the probability of error when +3 is transmitted.

The average BER will can be found by multiplying the individual BERs by the probabilities of the two symbols. In this question both symbols are equally probable (probability = 0.5). So the overall BER is $0.5 \times 0.091 + 0.5 \times 3.8 \times 10^{-3} \approx 4.75\%$.

Question 2

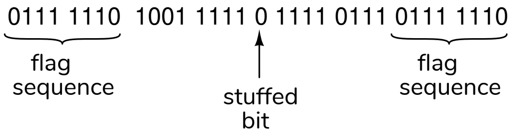
Write out the bits that would be transmitted if the bit sequence:

`1001 1111 1111 0111`

were to be framed using HDLC. Label the bits that you added and their purpose.

Answer

HDLC framing will add a flag sequence before and after the frame and will use bit stuffing to escape sequences of 5 consecutive one bits. The bits transmitted will be:



Question 3

A communication system uses the simple CRC algorithm described in the lecture notes to detect errors. The generator polynomial is $x^4 + x + 1$. The received message is `10111110`. Did the received message contain an error? Show your work.

Answer

We can divide the received message by the generator polynomial and if the remainder is zero then the received message is a multiple of the generator polynomial. This indicates that either there were no errors or the error pattern is a multiple of the generator polynomial.

The generator polynomial as a bit sequence is `10011`.

The division (using 0's and 1's instead of polynomials) is as follows:

```
            1010
      +---------
10011 | 10111110
        10011
        -----
         01001
         -----
          10011
          10011
          -----
           00000
           -----
            0000
```

The remainder is zero, so no errors were detected.