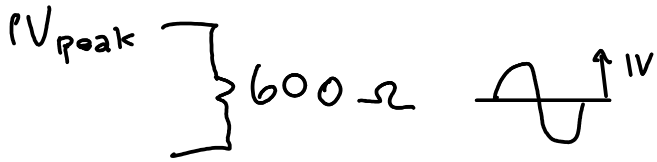
Noise

Exercise 1: A zero-mean sinusoidal signal is being transmitted over a noisy telephone channel. The voltage of the signal is measured with an oscilloscope and is found to have a peak voltage of 1V.

Nearby machinery adds noise to the line by induction. The voltage of this noise signal is measured with an RMS voltmeter as 100 mVrms. The line is connected to ("terminated with") a 600Ω load.

Why was an RMS voltmeter used to measure the noise? What is the signal power dissipated in the load? What is the noise power? What is the SNR?

Need to use an RMS voltmeter because noise is not a sine wave.



$$\frac{V^2}{R} = \frac{(1/\sqrt{2})^2}{600}$$

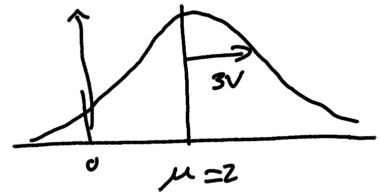
$$P_S = \frac{1/2}{600} W$$

$$P_N = \frac{(0.1)^2}{600} =$$

$$SNR = \frac{P_S}{P_N} = \frac{1/2}{(0.1)^2} = 50$$

$$10 \log 50 = 17 \text{ dB}$$

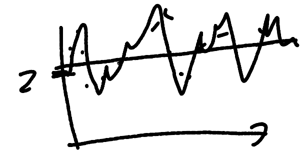
Exercise 2: Would you use AC or DC coupling to measure: (a) σ, (b) μ, and (c) the RMS power? Would you measure the average or RMS power in each case? What is the RMS power of the signal x if it has a mean (DC) value of $\mu = 2$ V and $\sigma = 3$ V?



	coupling to measure	measuring
σ	AC	rms
μ	DC	average
RMS power	AC+DC	rms

$\mu = 2$ V

$\sigma = 3$ V

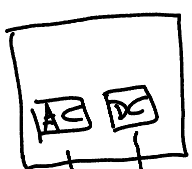


RMS power $= \sqrt{(DC)^2 + (AC)^2}$

$= \sqrt{2^2 + 3^2}$

$= \sqrt{13} \approx 3.6$

$\sqrt{4+9} = 13$



$-1, 2 \rightarrow$ average $= \frac{-1+2}{2}$

$= \frac{1}{2}$

→ average AND DC coupling

→ rms AND AC coupling

$1, 2 \rightarrow$ rms $= \frac{1^2 + 2^2}{2}$

$\sqrt{\frac{5}{2}} = 1.6$V

→ rms AND DC "

$-1, 2 \rightarrow$ rms $= \sqrt{\frac{(-1)^2 + 2^2}{2}} = \sqrt{\frac{5}{2}} = 1.6$ V.

=

Exercise 3: What are the units of t?

$\frac{V}{V}$ = no units

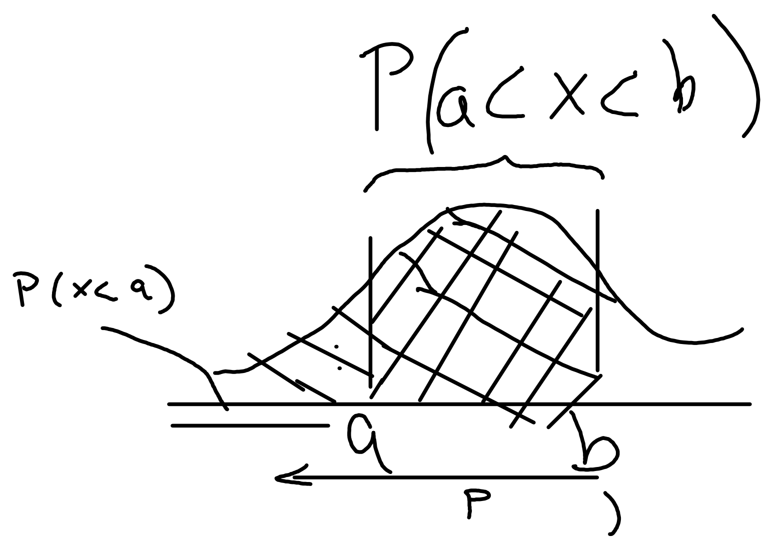
Exercise 4: The output of a noise source has a zero-mean Gaussian (normally) distributed output voltage. The (rms) output voltage is 100 mV. What fraction of the time does the output voltage exceed 200 mV? 300 mV? *Hint: the standard deviation (σ) of a zero-mean signal is the same as its RMS voltage.*

$$P(x < 200\,\text{mV}) = 97\%$$

$$P(x > 200\,\text{mV}) = 1 - 97\% = 3\%.$$

$$t = \frac{300 - 0}{100} = 3$$

Exercise 5: Mark examples of a and b on a Gaussian pdf and the three probabilities in the equation above.



Exercise 6: A signal $x(t)$ randomly switches between ± 1. What are the mean-square powers of $x(t)$, $y(t)$ and $x(t) + y(t)$ if: (a) $y(t) = x(t)$? (b) $y(t) = -x(t)$? (c) $y(t)$ randomly changes between ± 1 independently of $x(t)$? *Hint: work out the possible values and their probabilities.*

(x<b

(a)

y	x	P	
−1	−1	1/2	−2
−1	+1	0	0
+1	−1	0	0
+1	+1	1/2	+2

$4/2 + 4/2 = 4$

$V_{rms}^2(y) = 1$

$V_{rms}^2(x) = 1$

$V_{rms}^2(x+y) = 4$

(b)

y	x	P	
−1	−1	0	−2
−1	+1	1/2	0
+1	−1	1/2	0
+1	+1	0	+2

$V_{rms}^2(y) = 1$

$V_{rms}^2(x) = 1$

$V_{rms}^2(x+y) = 0$

$V_{rms}^2(y) = 1$

$V_{rms}^2(x) = 1$

c)

y	x	P	x+y
−1	−1	1/4	−2
−1	+1	1/4	0
+1	−1	1/4	0
+1	+1	1/4	+2

$$(-2)^2 \cdot \frac{1}{4} + (2)^2 \cdot \frac{1}{4} = 2$$