Exercise 11: A source generates four different messages. The first three have probabilities 0.125, 0.125, 0.25. What is the probability of the fourth message? How much information is transmitted by each message? What is the entropy of the source? What is the average information rate if 100 messages are generated every second? What if there were four equally-likely messages?

$$I_0 = -\log_2\left(\frac{1}{8}\right) = -(-3) = 3 \text{ bits}$$

$$I_3 = -\log_2\left(\frac{1}{2}\right) = 1 \text{ bit}$$

$$H = \frac{1}{8}\cdot 3 + \frac{1}{8}\cdot 3 + \frac{1}{4}\cdot 2 + \frac{1}{2}\cdot 1$$

$$= 1.75 \text{ bits/message}$$

Exercise 12: How long will it take to transfer 1 MByte at a rate of 10 kb/s?

$$\frac{8 \times 10^6 \text{ or } 8 \times 2^{20} \; ?}{10{,}000}$$

$$8 \times 10^2 = 800 \text{ s.}$$

Exercise 13: A communication system transmits one of the symbols above each microsecond. The probability of each symbol being transmitted is given above each symbol. What are the bit rate, the symbol rate, the information rate and the baud rate?

$$\text{bit rate} = \frac{\log_2(\#\text{symbols})}{\text{symbol duration}}$$

$$= \frac{\log_2(4)}{1 \times 10^{-6}} = 2 \times 10^{6} = 2\text{Mb/s}$$

$$\text{symbol rate} = \frac{1}{\text{symbol duration}} = \frac{1}{1 \times 10^{-6}} = 1\text{MHz}$$

$$\text{Entropy}\left(\frac{\text{bits}}{\text{message}}\right) = H = \sum_i P_i(-\log_2 P_i)$$

$$-\left(0.4\log_2(0.4) + 0.3\log_2(0.3) + 0.2\log_2(0.2) + 0.1\log_2(0.1)\right)$$

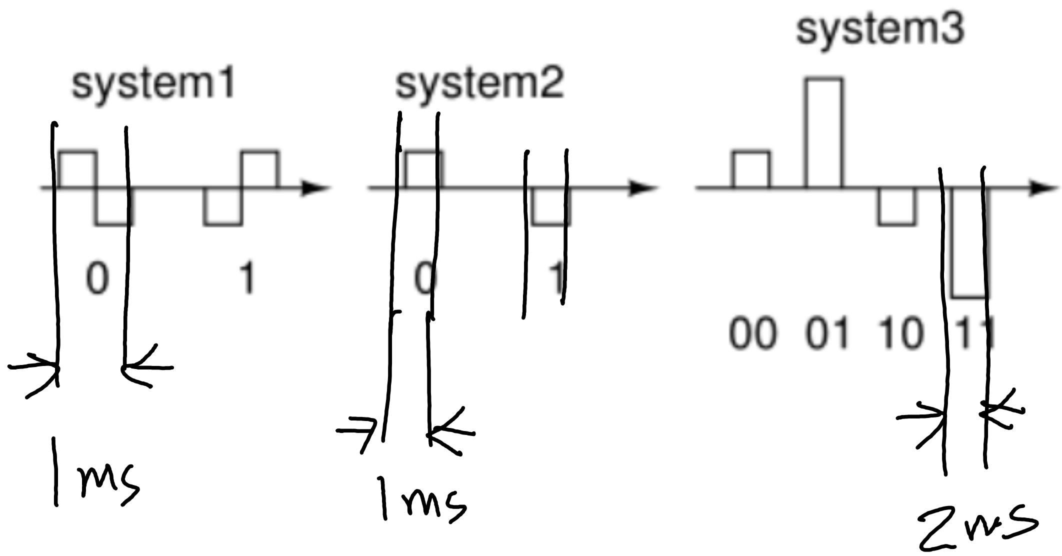
$= 1.82$ (see last years solutions).

$$\text{Information rate} = \frac{H}{\text{symbol duration}} = \frac{1.82}{1 \times 10^{-6}} = 1.82\text{ Mb/s}.$$

$$\text{baud rate} = \frac{1}{\text{minimum time between transitions}} = \frac{1}{0.5\,\mu s}.$$

$$= 2\ \text{MHz} \quad (\text{Mbaud?}).$$

Exercise 14: Another system, as shown above, encodes each bit using two pulses of opposite polarity (H-L for 0 and L-H for 1). A second system encodes bits using one pulse per bit (H for 0 and L for 1). A third system encodes two bits per pulse by using four different pulse levels (-3V for 00, -1V for 01, +1V for 10 and +3V for 11). Assuming each system transmits at 1000 bits per second, what are the baud rates in each case? How many different symbols are used by each system? What are the symbol rates? Assuming each symbol is equally likely, what are the information rates?



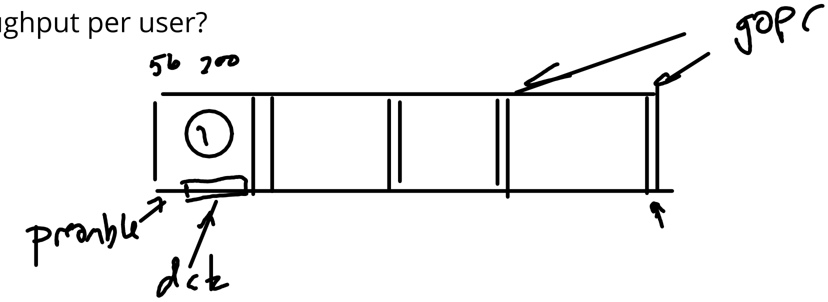
	2 kHz	1 kHz	500 Hz
# symbols	2	2	4
symbol rate	1 kHz	1 kHz	500 Hz
info. rate	$1/1ms$ = 1 kb/s	1 kb/s	$\frac{2 \text{ bits}}{2ms}$ = 1 kb/s.

Exercise 15: You receive 1 million frames, each of which contains 100 bits. By comparing the received frames to the transmitted ones you find that 56 frames had errors. Of these, 40 frames had one bit in error, 15 had two bit errors and one had three errors. What was the FER? The BER?

$$FER = \frac{56}{10^6} = 56 \times 10^{-6} = 5.6 \times 10^{-5}$$

$$BER = \frac{40 \times 1 + 15 \times 2 + 1 \times 3}{100 \cdot 10^6} = 73 \times 10^{-8} = 7.3 \times 10^{-7}$$

Exercise 16: A system transmits data at an (instantaneous) rate of 1 Mb/s in frames of 256 bytes. 200 of these bytes are data and the rest are overhead. The time available for transmission over the channel is shared equally between four users. A 200 μs gap must be left between each packet. What throughput does each user see? Now assume 10% of the frames are lost due to errors. What is the new throughput per user?



$$\text{Throughput} = \frac{\text{usable data}}{\text{total time}} = \frac{200 \text{ bytes}}{9 \text{ ms}} = 22.2 \text{ kbytes/s}$$

$$= 176 \text{ kb/s}.$$

$$\text{duration of 1 frame} = \frac{256 \frac{\text{bytes}}{\text{frame}} \cdot 8 \frac{\text{bits}}{\text{byte}}}{10^6 \text{ b/s}} = \frac{2048 \frac{\text{bits}}{\text{frame}}}{10^6 \text{ b/s}}$$

$$= 2048 \times 10^{-6} \text{ s/frame}.$$

$$= 2.05 \times 10^{-3} \text{ seconds}$$

plus gap = $= 2.25$ ms.

duration of 4 frames = $= 9$ ms

assuming no errors in retransmissions:

$$176 \text{ kb/s} \cdot \frac{10}{11}$$

← correct frames (10)

← total transmitted including retransmission (11)

$$\approx 160 \text{ kb/s}$$

Exercise 17: Plot some sample data rate versus time curves for these three types of sources. Can you think of some characteristics of a video source that might result in a variable bit rate when it is compressed? (*Hint: what types of redundancy are there in video?*)

Exercise 18: For each of the following communication systems identify the tolerance it is likely to have to errors and delay: a phone call between two people, “texting”, downloading a computer program, streaming a video over a computer network. What do you think might be the maximum tolerable delay for each?