Solutions to Quiz 5

Question 1

How many data bits are in each codeword of a (256,200) block code?

Answer

For an (n, k) block code $n = 256$ is the number of bits per codeword and $k = 200$ is the number of data bits ($n - k$ is the number of parity bits). Here $k = 200$ is the number of data bits in each codeword.

Question 2

What is the Hamming distance between the codewords **100111** and **111001**?

Answer

The Hamming distance is the number of bits that differ between two codewords. Comparing the differences between the two codewords:

```
100111
111001
------
011110
```

we see that the two codewords differ in four bit positions. Thus the Hamming distance is 4.

Question 3

A code has two codewords: **010101** and **101010**. How many errors is this code guaranteed to detect?

Answer

Since the code has only two codewords, we only have to find the Hamming distance between two codewords (in general we have to compare all pairs of codewords). In this case the minimum (and only) Hamming distance is 6:

```
010101
101010
------
111111
```

A block code with Hamming distance d_{min} is guaranteed to detect up to $d_{\text{min}} - 1$ errors. In this case the code can detect up to $6 - 1 = 5$ errors.

Question 4

A receiver using the code above receives the codeword **010100**. What codeword was most likely transmitted?

Comparing the received codeword to all valid codewords we find the distance to the first codeword is 1:

```
010101
010100
------
000001
```

and the distance to the second is 5:

```
101010
010100
------
111110
```

So a minimum distance decoder would select the first codeword as the one most likely to have been sent.

Answer

Question 5

How many errors is this code guaranteed to correct?

Answer

A block code with Hamming distance d_{min} is guaranteed to correct up to:

$$\lfloor \frac{d_{\text{min}} - 1}{2} \rfloor = \lfloor \frac{6-1}{2} \rfloor = \lfloor 2.5 \rfloor = 2$$

errors. In this case the code can correct up to 2 errors